

## **Radiation and Chemical reaction effects on MHD Flow over a Vertical Porous Plate with Heat Generation by Considering Double Diffusive Convection**

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### **ABSTRACT:**

The object of the present problem is to analyze the influence of a first-order homogeneous chemical reaction and thermal radiation on hydromagnetic free convection heat and mass transfer for a viscous fluid past a semi-infinite vertical moving porous plate embedded in a porous medium in the presence of thermal diffusion and heat generation. The fluid is considered to be a gray, absorbing-emitting but non-scattering medium, and the Rosseland approximation is used to describe the radiative heat flux in the energy equation. The plate moves with constant velocity in the direction of fluid flow while the free stream velocity is assumed to follow the exponentially increasing small perturbation law. The dimensionless governing equations for this investigation are solved analytically using two-term harmonic and non-harmonic functions. The effects of various parameters on the velocity, temperature and concentration fields as well as the skin-friction coefficient, Nusselt number and the Sherwood number are presented graphically.

**Keywords:** Chemical reaction, heat and mass transfer, thermal radiation, MHD, Porous medium.

### **1. INTRODUCTION**

Convective flows with simultaneous heat and mass transfer under the influence of a magnetic field and chemical reaction arise in many transport processes both naturally and artificially in many branches of science and engineering applications. This phenomenon plays an important role in the chemical industry, power and cooling industry for drying, chemical vapor deposition on surfaces, cooling of nuclear reactors and petroleum industries. Natural convection flow occurs frequently in nature. It occurs due to temperature differences, as well as due to concentration differences or the combination of these two, for example in atmospheric flows, there exists differences in water concentration and hence the flow is influenced by such concentration difference. Changes in fluid density gradients may be caused by non-reversible chemical reaction in the system as well as by the differences in molecular weight between values of the reactants and the products. Chemical reactions can be modeled as either homogeneous or heterogeneous processes. This depends on whether they occur at an interface or as a single phase volume reaction. A homogeneous reaction is one that occurs uniformly throughout a given phase. On the other hand, a heterogeneous reaction takes place in a restricted area or within the boundary of a phase. In most cases of chemical reactions, the reaction rate depends on the concentration of the species itself. A reaction is said to be of first order, if the rate of reaction is directly proportional to the concentration itself. For example, the formation of smog is a first order homogeneous reaction. Consider the emission of nitrogen dioxide from automobiles and other smoke-stacks. This nitrogen dioxide reacts chemically in the atmosphere with unburned hydrocarbons (aided by sunlight) and produces peroxyacetylnitrate, which forms an envelope which is termed photo-chemical smog. The study of heat and mass transfer with chemical reaction is of great practical importance in many branches of science and engineering. Das et al., [1] studied the effects of mass transfer flow past an impulsively started infinite vertical plate with constant heat flux and chemical reaction. Anjalidevi et.al, [2] studied effects of chemical reaction, heat and mass transfer on laminar flow along a semi-infinite horizontal plate. More recently, intensive studies have been carried out to investigate effects of chemical reaction on different flow types by Seddeek et al. [3] Salem & Abd El-Aziz, [4] Mohamed [5], Ibrahim et al.[6].

Convection flows in porous media has gained significant attention in recent years because of their importance in engineering applications such as geothermal systems, solid

matrix heat exchangers, thermal insulations, oil extraction and store of nuclear waste materials. These can also be applied to underground coal gasification, ground water hydrology, wall cooled catalytic reactors, energy efficient drying processes and natural convection in earth's crust. Detailed reviews of flow through and past porous media can be found in Nield and Benjan, [7]. Abel et al., [8] studied the two-dimensional boundary layer problem on mixed convection of an incompressible visco-elastic fluid immersed in a porous medium over a stretching sheet. Ali, [9] analyzed the effect of lateral mass flux on natural convection boundary layer induced by heated vertical plate embedded in a saturated porous medium.

The use of magnetic field that influences heat generation/absorption process in electrically conducting fluid flows has many engineering applications. For example, many metallurgical processes which have involve cooling of continuous strips or filaments, which are drawn through a quiescent fluid. The properties of the final product depend to a great extent on the rate of cooling. The rate of cooling and therefore, the desired properties of the end product can be controlled by the use of electrically conducting fluids and the applications of the magnetic fields are studied by Vajravelu et.al. [10]. Many works have been reported on flow, heat and mass transfer of electrically conducting fluids over semi-infinite/infinite plates/stretching surfaces in the presence of magnetic field investigated by Chamkha et.al [11], Shateyi et al. [12], Makinde et.al., [13], B'eg et al. [14], Pal & Talukdar [15], Makinde & Aziz [16].

The study of heat generation or absorption in moving fluids is important in problems dealing with chemical reactions and those concerned with dissociating fluids. Heat generation effects may alter the temperature distribution and this in turn can affect the particle deposition rate in nuclear reactors, electronic chips and semi conductor wafers. Although exact modeling of internal heat generation or absorption is quite difficult, some simple mathematical models can be used to express its general behaviour for most physical situations. Heat generation or absorption can be assumed to be constant, space-dependent or temperature-dependent. Crepeau et al. [17] have used a space-dependent exponentially decaying heat generation or absorption in their study on flow and heat transfer from a vertical plate. Several interesting computational studies of reactive MHD boundary layer flows with heat and mass transfer in the presence of heat generation or absorption have studied by Patil et al.[18]; Salem et.al, [19], Samad et al., [20]. The effect of radiation on MHD flow, heat and mass transfer become more important industrially. Many processes in engineering areas occur at high temperature and knowledge of radiation heat transfer becomes a very important for the design of the pertinent equipment. The quality of the final product depends to a great extent on the heat controlling factors, and the knowledge of radiative heat transfer in the system can lead to a desired product with sought qualities. Different researches have been forwarded to analyze the effects of thermal radiation on different flows are studied by Cortell, [21], Bataller, [22], Ibrahim et al. [23], Shateyi [24], Shateyi and Motsa, [25], Aliakba et al. [26], Hayat et.al [27], Cortell [28]. Kesavaiah et al [29] have analyzed the effects of the chemical reaction and radiation absorption on unsteady MHD convective heat and mass transfer flow past a semi-infinite vertical permeable moving plate embedded in porous medium with heat source and suction. Ahammad et al [30] represents the effects of variable chemical reaction and variable electric conductivity on free convective heat and mass transfer flow along an inclined stretching sheet with variable heat of Dufour and Soret effects. Bhaskar Reddy et al [31] have studied the radiation and mass transfer effects on MHD free convection flow along a stretching surface with viscous dissipation and heat generation. Singh et al [32] presented the heat transfer in a second-grade fluid over an exponentially stretching sheet through porous medium with thermal radiation and elastic deformation under the effect of magnetic field. Sridhar Sarma et al [33] solve the problem of the combined effect of chemical reaction, thermal radiation on steady free convection and mass transfer flow in a porous medium considering Soret and Dufour effects. Raju et al [34] have

analyzed the Soret effects due to natural convection in a non-Newtonian fluid flow in porous medium with heat and mass transfer. Vedavathi et al [35] have analyzed radiation and mass transfer effects on unsteady MHD convective flow past and infinite vertical plate with Dufour and Soret effects.

In spite of all the previous studies, the unsteady MHD free convection heat and mass transfer for a heat generation/absorption with radiation absorption in the presence of a reacting species over an infinite permeable plate has received little attention. Hence, the main objective of this chapter is to investigate the effects of thermal radiation, chemical reaction, and heat source/sink parameter of an electrically conducting fluid past an infinite vertical porous plate subjected to variable suction. The plate is assumed to be embedded in a uniform porous medium and moves with a constant velocity in the flow direction in the presence of a transverse magnetic field. The governing equations of motion are solved analytically by using a regular perturbation technique. The behaviour of velocity, temperature, concentration, skin-friction, Nusselt number and Sherwood number for different values of thermo-physical parameters have been computed and the results are presented graphically and discussed qualitatively.

## 2. FORMATION OF THE PROBLEM

Two dimensional unsteady flow of a laminar, incompressible, viscous, electrically conducting and heat generation/absorption fluid past a semi infinite vertical moving porous plate embedded in a uniform porous medium and subjected to a uniform transverse magnetic field in the presence of a pressure gradient has been considered with double-diffusive free convection, thermal diffusion, chemical reaction, and thermal radiation effects. According to the coordinate system the  $x'$  - axis is taken along the porous plate in the upward direction and  $y'$  - axis normal to it. The fluid is assumed to be a gray, absorbing, emitting but non-scattering medium. The radiative heat flux in the  $x'$  - direction is considered negligible in comparison with that in the  $y'$  - direction. It is assumed that there is no applied voltage of which implies the absence of an electric field. The transversely applied magnetic field and magnetic Reynolds number are very small and hence the induced magnetic field is negligible. Viscous and Darcy resistance terms are taken into account the constant permeability porous medium. The MHD term is derived from an order of magnitude analysis of the full Navier - Stokes equation. It is assumed here that the whole size of the porous plate is significantly larger than a characteristic microscopic length scale of the porous medium. The chemical reactions are taking place in the flow and all thermo physical properties are assumed to be constant of the linear momentum equation which is approximation. The fluid properties are assumed to be constants except that the influence of density variation with temperature and concentration has been considered in the body force term. Due to the semi infinite plate surface assumption furthermore, the flow variables are functions of  $y'$  and  $t'$  only. The governing equation for this investigation is based on the balances of mass, linear momentum, energy, and concentration species. Taking into consideration the assumptions made above, these equations can be written in Cartesian frame of reference, as follows:

$$\frac{\partial v'}{\partial y'} = 0 \quad (1)$$

$$\frac{\partial u'}{\partial t'} + v' \frac{\partial u'}{\partial y'} = -\frac{1}{\rho} \frac{\partial p'}{\partial x'} + \nu \frac{\partial^2 u'}{\partial y'^2} + g\beta(T' - T_\infty) + g\beta^*(C' - C_\infty) - \frac{\sigma B_0^2}{\rho} u' - \frac{\nu}{k'} u' \quad (2)$$

$$\frac{\partial T'}{\partial t'} + v' \frac{\partial T'}{\partial y'} = \frac{k}{\rho c_p} \frac{\partial^2 T'}{\partial y'^2} - \frac{1}{\rho c_p} \frac{\partial q'_r}{\partial y'} + \frac{Q_0}{\rho c_p} (T' - T'_\infty) \quad (3)$$

$$\frac{\partial C'}{\partial t'} + v' \frac{\partial C'}{\partial y'} = D_M \frac{\partial^2 C'}{\partial y'^2} + D_T \frac{\partial^2 T'}{\partial y'^2} - R' (C' - C'_\infty) \quad (4)$$

The boundary conditions for the velocity, temperature, and concentration fields are given as follows:

$$u' = u'_p, T' = T'_w + \varepsilon(T'_w - T'_\infty)e^{n't'}, C' = C'_w + \varepsilon(C'_w - C'_\infty)e^{n't'} \quad \text{at } y' = 0$$

$$u' = U'_\infty = U_0(1 + \varepsilon e^{n't'}), T' \rightarrow T'_\infty, C' \rightarrow C'_\infty \quad \text{as } y' \rightarrow \infty \quad (5)$$

where  $u', v'$  are the velocity components in  $x', y'$  directions respectively,  $t'$  – the time,  $p'$  – the pressure,  $\rho$  – the fluid density,  $g$  – the acceleration due to gravity,  $\beta$  and  $\beta^*$  – the thermal and concentration expansion coefficients respectively,  $K'$  – the permeability of the porous medium,  $T'$  – the temperature of the fluid in the boundary layer,  $\nu$  – the kinematic viscosity,  $\sigma$  – the electrical conductivity of the fluid,  $T'_\infty$  – the temperature of the fluid far away from the plate,  $C'$  – the species concentration in the boundary layer,  $C'_\infty$  – the species concentration in the fluid far away from the plate,  $B_0$  – the magnetic induction,  $\alpha$  – the fluid thermal diffusivity,  $q'_r$  – the radiative heat flux,  $c_p$  – specific heat at constant,  $D_M$  – the coefficient of chemical molecular diffusivity,  $D_T$  – the coefficient of thermal diffusivity,  $R'$  – the reaction rate constant. The term  $Q_0(T' - T'_\infty)$  is assumed to be amount of heat generated or absorbed per unit volume  $Q_0$  is a constant, which may take on either positive or negative values. When the wall temperature  $T'_w$  exceeds the free stream temperature  $T'_\infty$ , the source term  $Q_0 > 0$  and heat sink when  $Q_0 < 0$ . The magnetic field and viscous dissipations are neglected in this study. It is assumed that the porous plate moves with a constant velocity  $u'_p$  in the direction of fluid flow, and the free stream velocity  $U'_\infty$  follows the exponentially increasing small perturbation law. In addition, it is assumed that the temperature and concentration at the wall as well as the suction velocity are exponentially varying with time.  $u'_p$  is the plate velocity,  $T'_w$  and  $C'_w$  are the wall dimensional temperature and concentration, respectively,  $C'_\infty$  is the free stream dimensional concentration.  $U_0$  and  $n'$  are constants.

By using the Rosseland approximation, the radiative flux vector  $q_r$  can be written as:

$$q'_r = \frac{4\sigma^*}{3k'_1} \frac{\partial T'^4}{\partial y'} \quad (6)$$

where,  $\sigma^*$  and  $k'_1$  are respectively the Stefan-Boltzmann constant and the mean absorption coefficient. We assume that the temperature difference within the flow is sufficiently small such that  $T'^4$  may be expressed as a linear function of the temperature. This is accomplished by expanding in a Taylor series about the free stream temperature  $T'_\infty$  and neglecting higher order terms, thus

$$T'^4 \cong 4T'^3_\infty T' - 3T'^4_\infty \quad (7)$$

The energy equation after substitution of equations (6) and (7) can be written as:

$$\frac{\partial T'}{\partial t'} + v' \frac{\partial T'}{\partial y'} = \frac{k}{\rho c_p} \frac{\partial^2 T'}{\partial y'^2} + \frac{16\sigma^* T'^3_\infty}{3\rho c_p k'_1} \frac{\partial^2 T'}{\partial y'^2} + \frac{Q_0}{\rho c_p} (T' - T'_\infty) \quad (8)$$

From Equation (1), it is clear that suction velocity at the plate is either a constant or function of time only. Hence the suction velocity normal to the plate is assumed in the form

$$v' = -V_0 \left(1 + \varepsilon A \exp^{nt'}\right) \quad (9)$$

where  $A$  is a real positive constant, and  $\varepsilon$  is small such that  $\varepsilon \ll 1, \varepsilon A \ll 1$ , and  $v_0$  is a non-zero positive constant, the negative sign indicates that the suction is towards the plate.

Outside the boundary layer, Equation (2) gives

$$-\frac{1}{\rho} \frac{\partial p'}{\partial x'} = \frac{dU_\infty}{dt'} + \frac{v}{K'} U_\infty' + \frac{\sigma}{\rho} B_0^2 U_\infty' \quad (10)$$

In order to write the governing equations and the boundary conditions in dimensionless form, the following non-dimensional quantities are introduced.

$$\begin{aligned} u &= \frac{u'}{U_0}, v = \frac{v'}{V_0}, y = \frac{V_0 y'}{\nu}, U_\infty = \frac{U_\infty'}{U_0}, U_p = \frac{u'_p}{U_0}, t = \frac{t' V_0^2}{\nu}, R = \frac{4\sigma^* T_\infty'^3}{k_1 k}, \\ \theta &= \frac{T' - T_\infty'}{T'_w - T_\infty'}, \phi = \frac{C' - C_\infty'}{C'_w - C_\infty'}, n = \frac{n' \nu}{V_0^2}, K = \frac{K' V_0^2}{\nu^2}, \text{Pr} = \frac{\nu \rho c_p}{k} = \frac{\nu}{\alpha}, \\ Sc &= \frac{\nu}{D_M}, M = \frac{\sigma B_0^2 \nu}{\rho V_0^2}, Gr = \frac{\nu \beta g (T'_w - T_\infty')}{U_0 V_0^2}, Gm = \frac{\nu \beta^* g (C'_w - C_\infty')}{U_0 V_0^2}, \\ \eta &= \frac{\nu Q_0}{\rho V_0^2 c_p}, So = \frac{D_T (T'_w - T_\infty')}{\nu (C'_w - C_\infty')}, \delta = \frac{R' \nu}{V_0^2}, Q = \frac{\nu Q_0}{\rho c_p V_0^2}, N = M + \frac{1}{K} \end{aligned} \quad (11)$$

In view of Equations (8), (9) (10) and (11), Equations (2), (3) and (4) can be reduce to the following dimensionless form.

$$\frac{\partial u}{\partial t} - \left(1 + \varepsilon A e^{nt}\right) \frac{\partial u}{\partial y} = \frac{dU_\infty}{dt} + \frac{\partial^2 u}{\partial y^2} + G_r \theta + G_m \phi + N (U_\infty - u) \quad (12)$$

$$\frac{\partial \theta}{\partial t} - \left(1 + \varepsilon A e^{nt}\right) \frac{\partial \theta}{\partial y} = \frac{1}{\text{Pr}} \left(1 + \frac{4R}{3}\right) \frac{\partial^2 \theta}{\partial y^2} + Q \theta \quad (13)$$

$$\frac{\partial \phi}{\partial t} - \left(1 + \varepsilon A e^{nt}\right) \frac{\partial \phi}{\partial y} = \frac{1}{Sc} \frac{\partial^2 \phi}{\partial y^2} + S_o \frac{\partial^2 \theta}{\partial y^2} - \delta \phi \quad (14)$$

The corresponding dimensionless boundary conditions are

$$\begin{aligned} u &= U_p, \quad \theta = 1 + \varepsilon e^{nt}, \quad \phi = 1 + \varepsilon e^{nt} & \text{at } y = 0 \\ u &\rightarrow U_\infty = (1 + \varepsilon e^{nt}), \quad \theta \rightarrow 0, \quad \phi \rightarrow 0 & \text{as } y \rightarrow \infty \end{aligned} \quad (15)$$

### 3. SOLUTION OF PROBLEM

In order to reduce the above system of partial differential equations to a system of ordinary equations in dimensionless form, we may represent the velocity, temperature and concentration as

$$u(y, t) = u_0(y) + \varepsilon e^{nt} u_1(y) + O(\varepsilon^2) + \dots \quad (16)$$

$$\theta(y, t) = \theta_0(y) + \varepsilon e^{nt} \theta_1(y) + O(\varepsilon^2) + \dots \quad (17)$$

$$\phi(y, t) = \phi_0(y) + \varepsilon e^{nt} \phi_1(y) + O(\varepsilon^2) + \dots \quad (18)$$

Substituting (16), (17) and (18) in Equations (12) - (14) and equating harmonic and non-harmonic terms, and neglecting the higher order terms of  $O(\varepsilon^2)$ , we obtain

$$u_0'' + u_0' - Nu_0 = -[Gr\theta_0 + Gm\phi_0 + N] \quad (19)$$

$$u_1'' + u_1' - (N + n)u_1 = -[Gr\theta_1 + Gm\phi_1 + (N + n) + Au_0'] \quad (20)$$

$$\theta_0'' + F_1\theta_0' + F_1\theta_0 = 0 \quad (21)$$

$$\theta_1'' + F_1\theta_1' + F_1(Q - n)\theta_1 = 0 \quad (22)$$

$$\phi_0'' + Sc\phi_0' - Sc\delta\phi_0 = -SoSc\theta_0'' \quad (23)$$

$$\phi_1'' + Sc\phi_1' - (n + \delta)Sc\phi_1 = -ASc\phi_0' - SoSc\theta_1'' \quad (24)$$

where the primes denote the differentiation with respect to  $y$ .

The corresponding boundary conditions can be written as

$$u_0 = U_p, \quad u_1 = 0, \quad \theta_0 = 1, \quad \theta_1 = 1, \quad \phi_0 = 1, \quad \phi_1 = 1 \quad \text{at} \quad y = 0 \quad (25)$$

$$u_0 = 1, \quad u_1 = 1, \quad \theta_0 \rightarrow 0, \quad \theta_1 \rightarrow 0, \quad \phi_0 \rightarrow 0, \quad \phi_1 \rightarrow 0 \quad \text{as} \quad y \rightarrow \infty$$

The analytical solutions of equations (19) – (24) with satisfying the boundary conditions (25) are given by

$$u_0 = A_{16}e^{-m_4y} + A_{14}e^{-m_1y} + A_{15}e^{-m_3y} + 1 \quad (26)$$

$$u_1 = 1 - A_{22}e^{-m_1y} - A_{23}e^{-m_2y} - A_{24}e^{-m_3y} - A_{25}e^{-m_4y} + A_{26}e^{-m_5y} + A_{27}e^{-m_6y} \quad (27)$$

$$\theta_0 = e^{-m_1y} \quad (28)$$

$$\theta_1 = A_2e^{-m_2y} + A_1e^{-m_1y} \quad (29)$$

$$\phi_0 = A_4e^{-m_3y} + A_3e^{-m_1y} \quad (30)$$

$$\phi_1 = A_{11}e^{-m_4y} + A_8e^{-m_3y} + A_9e^{-m_1y} + A_{10}e^{-m_2y} \quad (31)$$

In view of the above solutions, the velocity, temperature and concentration distributions in the boundary layer become

$$u(y, t) = A_{16}e^{-m_4y} + A_{14}e^{-m_1y} + A_{15}e^{-m_3y} + 1 + \varepsilon \left[ \begin{array}{l} 1 - A_{22}e^{-m_1y} - A_{23}e^{-m_2y} - A_{24}e^{-m_3y} \\ -A_{25}e^{-m_4y} + A_{26}e^{-m_5y} + A_{27}e^{-m_6y} \end{array} \right] e^{nt} \quad (32)$$

$$\theta(y, t) = e^{-m_1y} + \varepsilon [A_2e^{-m_2y} + A_1e^{-m_1y}] e^{nt} \quad (33)$$

$$\phi = A_4e^{-m_3y} + A_3e^{-m_1y} + \varepsilon [A_{11}e^{-m_4y} + A_8e^{-m_3y} + A_9e^{-m_1y} + A_{10}e^{-m_2y}] e^{nt} \quad (34)$$

It is now important to calculate the physical quantities of primary interest, which are the local wall shear stress, the local surface heat, and mass flux. Given the velocity field in the boundary layer, we can now calculate the local wall shear stress (i.e., skin-friction) is given by

$$\tau_w^* = \mu \left( \frac{\partial u'}{\partial y'} \right)_{y'=0}, \quad (35)$$

and in dimensionless form, we obtain

$$C_f = \frac{\tau_w^*}{\rho U_0 V_0} = \left( \frac{\partial u}{\partial y} \right)_{y=0} = -m_4A_{16} - m_1A_{14} - m_3A_{15} + \varepsilon \left[ \begin{array}{l} m_1A_{22} + m_2A_{23} + m_3A_{24} \\ +m_4A_{25} - m_5A_{26} - m_6A_{27} \end{array} \right] e^{nt} \quad (36)$$

Knowing the temperature field, it is interesting to study the effect of the free convection and radiation on the rate of heat transfer  $q_w^*$ . This is given by

$$q_w^* = -k \left( \frac{\partial T'}{\partial y'} \right)_{y'=0} - \frac{4\sigma^*}{3k_1^*} \left( \frac{\partial T'^4}{\partial y'} \right)_{y'=0} \quad (37)$$

By using equation (7), we can write equation (37) as follow

$$q_w^* = - \left( k + \frac{16\sigma^* T_\infty'}{3k_1^*} \right) \left( \frac{\partial T'}{\partial y'} \right)_{y'=0} \quad (38)$$

which is written in dimensionless form as;

$$q_w^* = - \frac{k(T_w' - T_\infty') V_0}{\nu} \left( 1 + \frac{4R}{3} \right) \left( \frac{\partial \theta}{\partial y} \right)_{y=0} \quad (39)$$

The dimensionless local surface heat flux (i.e., Nusselt number) is obtained as

$$\begin{aligned} Nu = \frac{xq_w^*}{k(T_w' - T_\infty')} &\Rightarrow Nu Re_x^{-1} = - \left( 1 + \frac{4R}{3} \right) \left( \frac{\partial \theta}{\partial y} \right)_{y=0} \\ &= - \left( 1 + \frac{4R}{3} \right) \left[ -m_1 + \varepsilon (-A_2 m_1 - A_1 m_2) e^{nt} \right] \end{aligned} \quad (40)$$

where,  $Re_x = \frac{V_0 x}{\nu}$  is the Reynolds number

The definition of the local mass flux and the local Sherwood number are respectively given by

$$q_w^* = -D \left( \frac{\partial C'}{\partial y'} \right)_{y'=0} \quad (41)$$

$$Sh_x = \frac{j_w x}{D(C_w' - C_\infty')} \quad (42)$$

with the help of these equations, one can write

$$Sh_x Re_x^{-1} = - \left( \frac{\partial \phi}{\partial y} \right)_{y=0} = -A_4 m_3 - A_3 m_1 + \varepsilon [-A_{11} m_1 - A_8 m_3 - A_9 m_1 - A_{10} m_2] e^{nt} \quad (43)$$

#### 4. RESULTS AND DISCUSSION

The formulation of the effects of chemical reaction, thermal diffusion, heat source and thermal radiation on MHD convective flow and mass transfer of an incompressible, viscous fluid along a semi infinite vertical porous moving plate in a porous medium has been performed in the receding sections. This enables us to carry out the numerical calculations for the distribution of the velocity, temperature and concentration across the boundary layer for various values of the parameters. In the present study we have chosen  $A=0.5, t=1.0, n=0.1, Up=0.5$ , and  $\varepsilon=0.1$ , while  $R, \delta, Q, So, Gr, Gm, M, Pr, Sc$  and  $K$  are varied over a range, which are listed in the figure legends. Also, the boundary condition for  $y \rightarrow \infty$  is replaced by where  $y_{max}$  is a sufficiently large value of  $y$  where the velocity profile  $u$  approaches to the relevant free stream velocity.

For different values of radiation parameter  $R$ , the velocity profiles are plotted in Fig.1. Here we find that, as the value of  $R$  increases the velocity increases, with an increasing in the flow boundary layer thickness. Thus, thermal radiation enhances convective flow. For different values of the magnetic field parameter  $M$ , the velocity profiles are plotted in Fig.2. It is obvious that the effect of increasing values of  $M$  parameter results in decreasing velocity distribution across the boundary layer because of the application of transfer magnetic field will result a restrictive type force (Lorentz force) similar to drag force which tends to resist the fluid flow and thus reducing its velocity.

The influences of chemical reaction parameter  $\delta$  on the velocity profiles across the boundary layer are presented in Fig.3. We see that the velocity distribution across the boundary layer decreases with increasing of  $\delta$ . The effect of heat generation  $Q$  on the velocity profiles is shown in Fig.4. From this figure we see that the heat is generated the buoyancy force increases which induce the flow rate to increase giving rise to the increase in the velocity profiles.

The effects of Soret number  $So$  on the velocity profiles is shown in Fig.5. From this figure we see that velocity profiles increase with an increasing of Soret number  $So$  from which we conclude that the fluid velocity rises due to greater thermal diffusion.

The velocity profiles for different values of Grashof number  $Gr$  are described in Fig.6. It is observed that an increasing in  $Gr$  leads to a rise in the values of velocity. Here the Grashof number represents the effects of the free convection currents. Physically,  $Gr > 0$  means heating of the fluid of cooling of the boundary surface  $Gr < 0$  means cooling of the fluid of heating of the boundary surface and  $Gr = 0$  corresponds to the absence of free convection current. In addition, the curves show that the peak value of velocity increases rapidly near the wall of the porous plate as Grashof number increases, and then decays to the relevant free stream velocity.

The velocity profiles for different values of Solutal Grashof number  $Gc$  are described in Fig.7. It is observed that an increasing in  $Gc$  leads to a rise in the values of velocity. In addition, the curves show that the peak value of velocity increases rapidly near the wall of the porous plate as Solutal Grashof number increases, and then decays to the relevant free stream velocity.

Fig.8 shows the velocity profiles across the boundary layer for different values of Prandtl number  $Pr$ . The results show that the effect of increasing values of  $Pr$  results in a decreasing the velocity.

Fig. 9 shows the velocity profiles for different values of the permeability  $K$ . Clearly as  $K$  increases the peak value of velocity tends to increase. These results could be very useful in deciding the applicability of enhanced oil recovery in reservoir engineering.

Typical variation of the temperature profiles along the span wise coordinate  $y$  are shown in Fig.10 for different values of Prandtl number  $Pr$ . The results show that an increase of Prandtl number results in a decreasing the thermal boundary layer thickness and more uniform temperature distribution across the boundary layer. The reason is that smaller values of  $Pr$  are equivalent to increasing the thermal conductivities, and therefore, heat is able to differ away from the heated surface more rapidly than for higher values of  $Pr$ . Hence, the boundary layer is thicker and the rate of heat transfer is reduced, for gradient have been reduced.

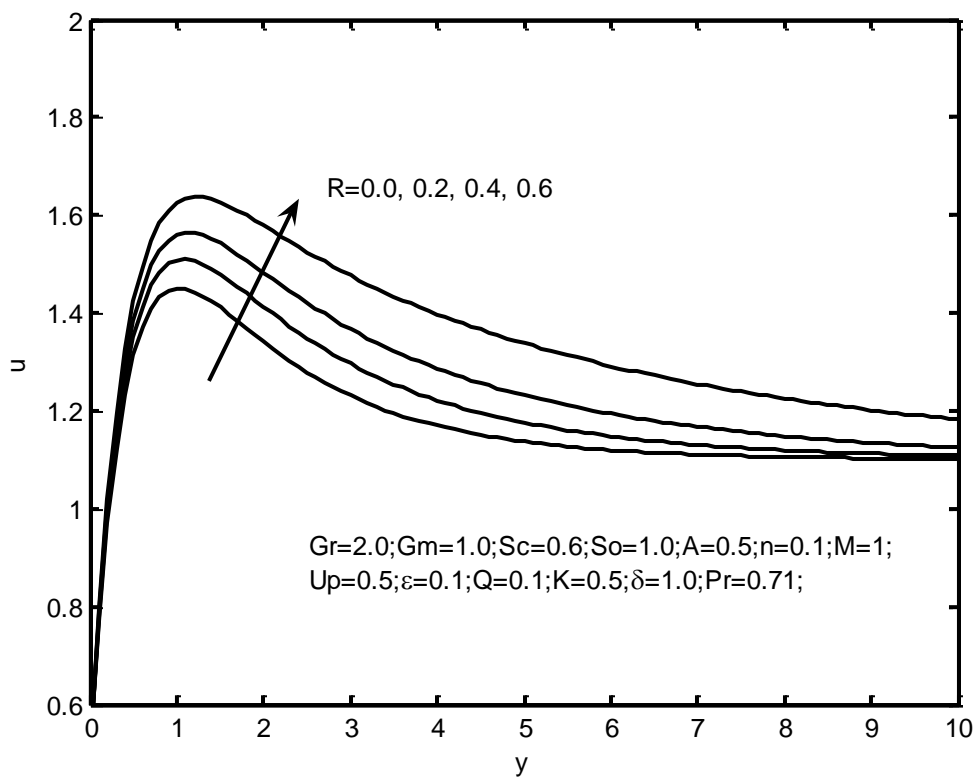
The effects of radiation parameter  $R$  on the temperature profiles are presented in Fig.11. From this figure we observe that, as the value of  $R$  increases the temperature profiles increases, with an increasing in the thermal boundary layer thickness.

Fig.12 shows the variation of temperature profiles for different values of  $Q$ . It is seen from this figure that temperature profiles increase with an increasing of heat generation parameter  $Q$ .

For different values of the chemical reaction parameter  $\delta$ , the concentration profiles plotted in Fig.13. It is obvious that the influence of increasing values of  $\delta$ , the concentration distribution across the boundary layer decreases.

Fig.14 shows the concentration profiles across the boundary layer for various values of Schmidt number  $Sc$ . The figure shows that an increasing in  $Sc$  results in a decreasing the concentration distribution, because the smaller values of  $Sc$  are equivalent to increasing the chemical molecular diffusivity.





**Fig.1. Velocity profiles for different values of radiation parameter (R).**

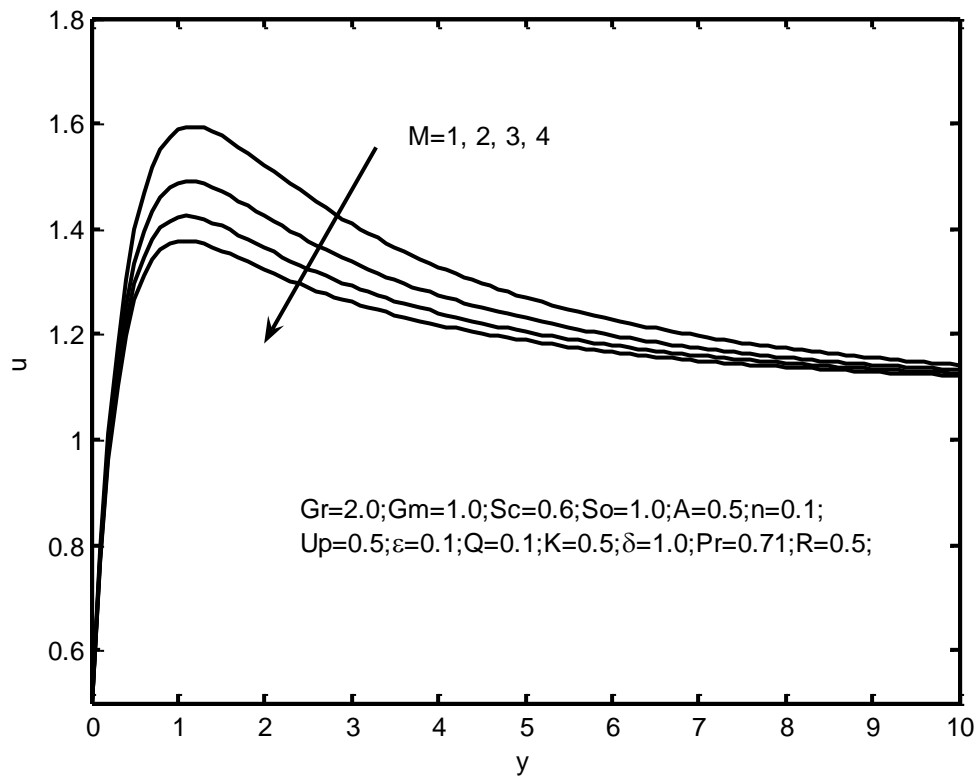


Fig.2. Velocity profiles for different values of magnetic parameter ( $M$ ).

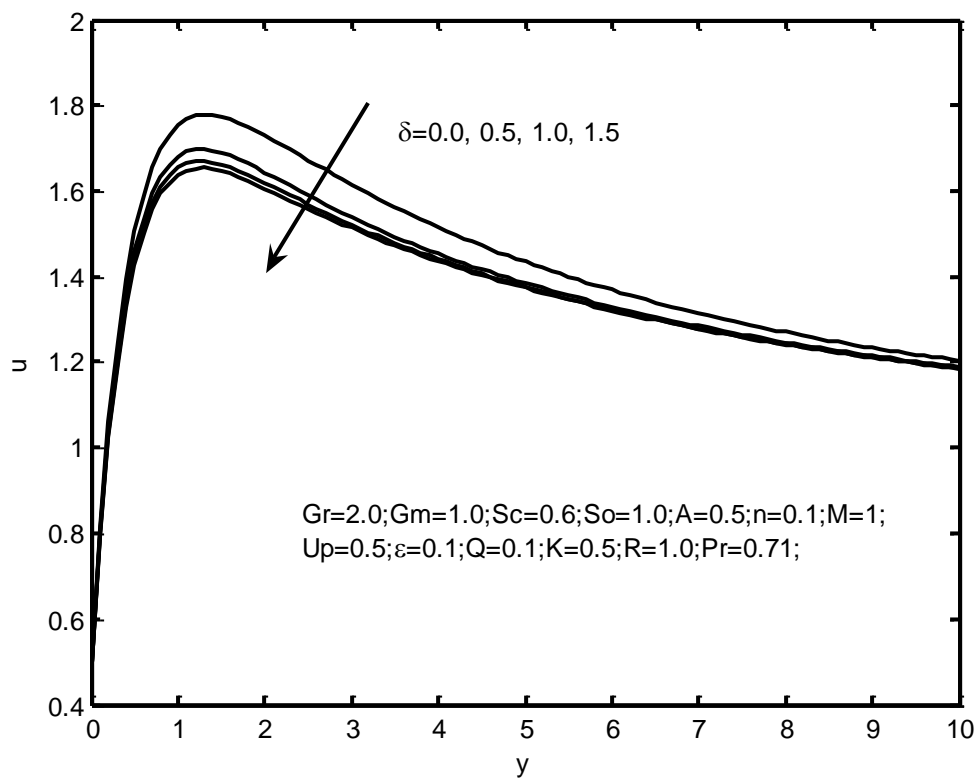
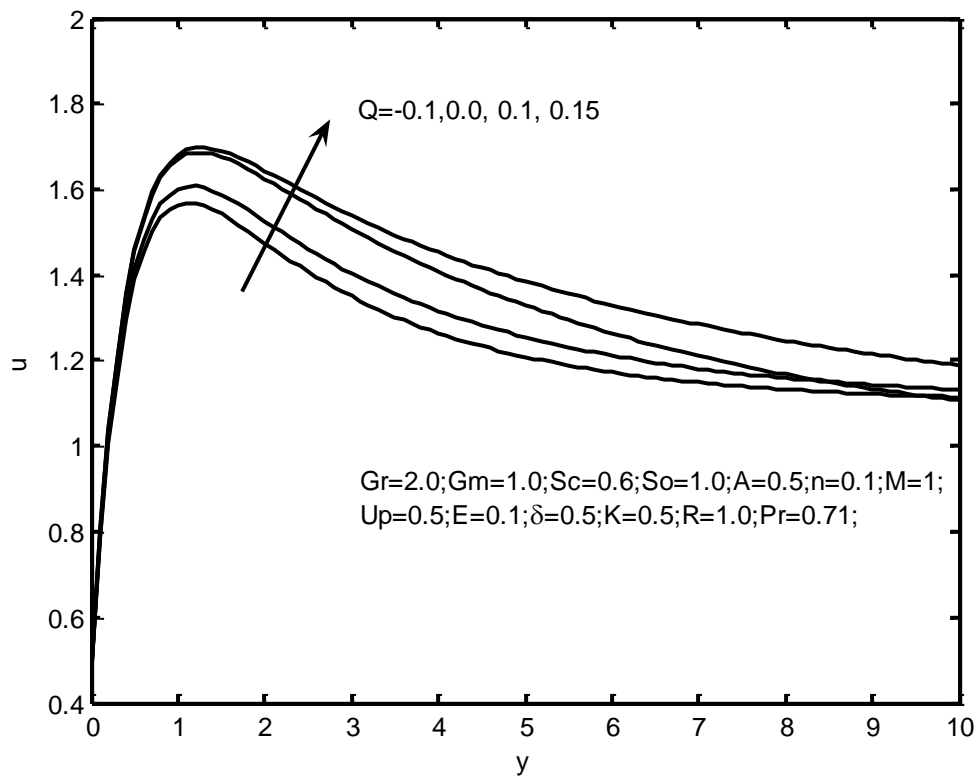
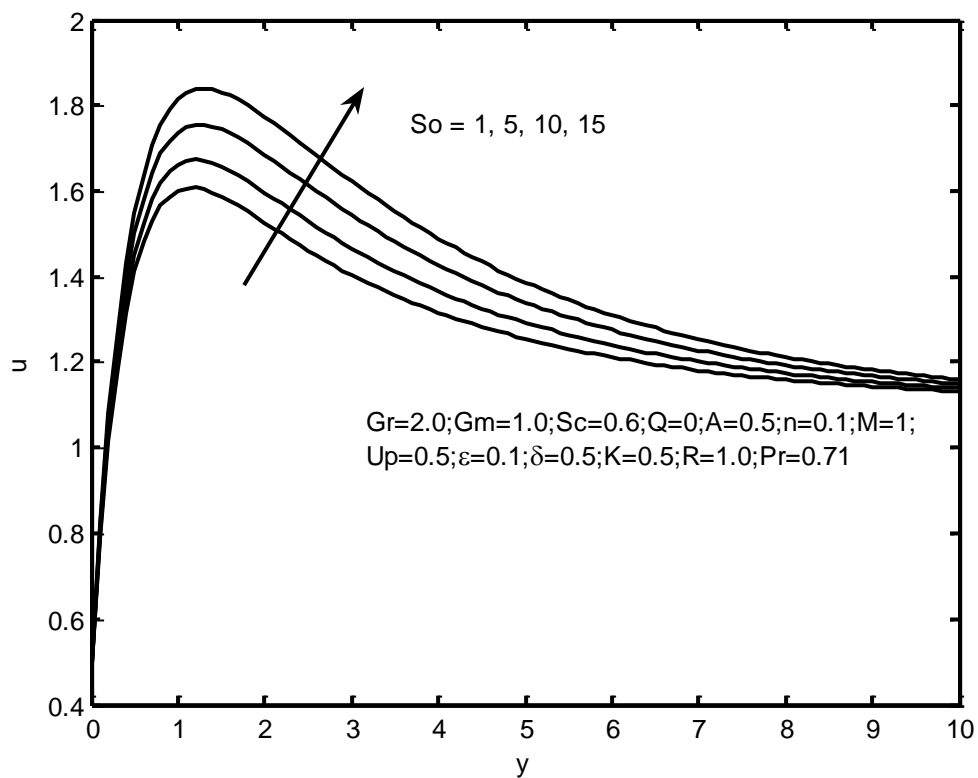


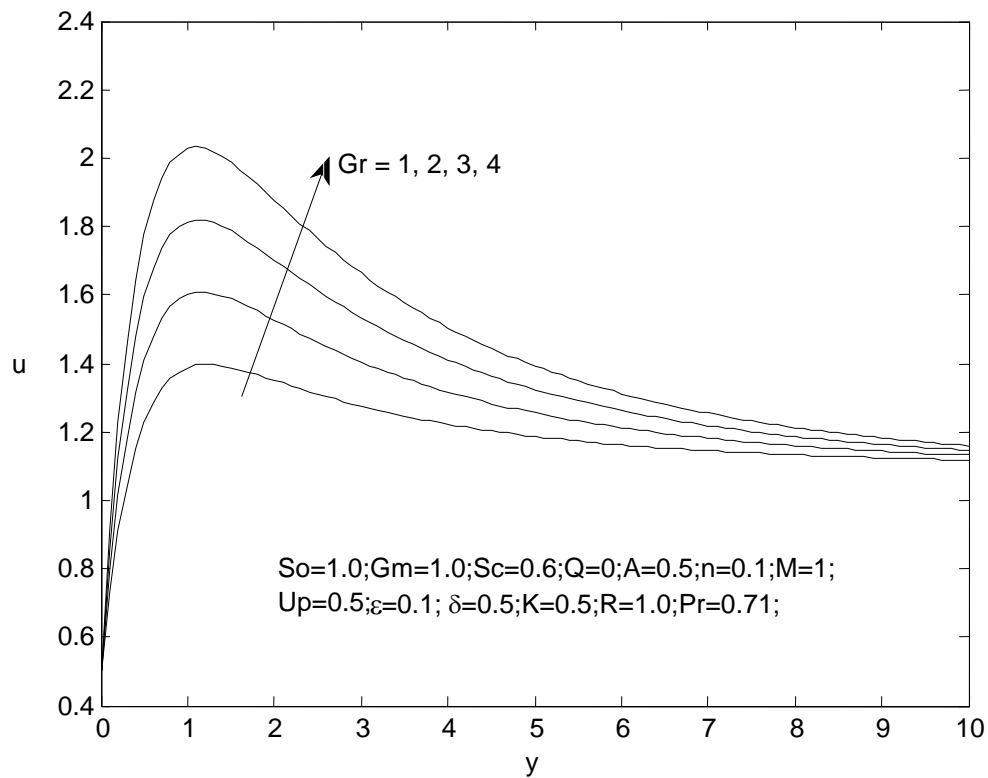
Fig.3. Velocity profiles for different values of chemical reaction parameter ( $\delta$ ).



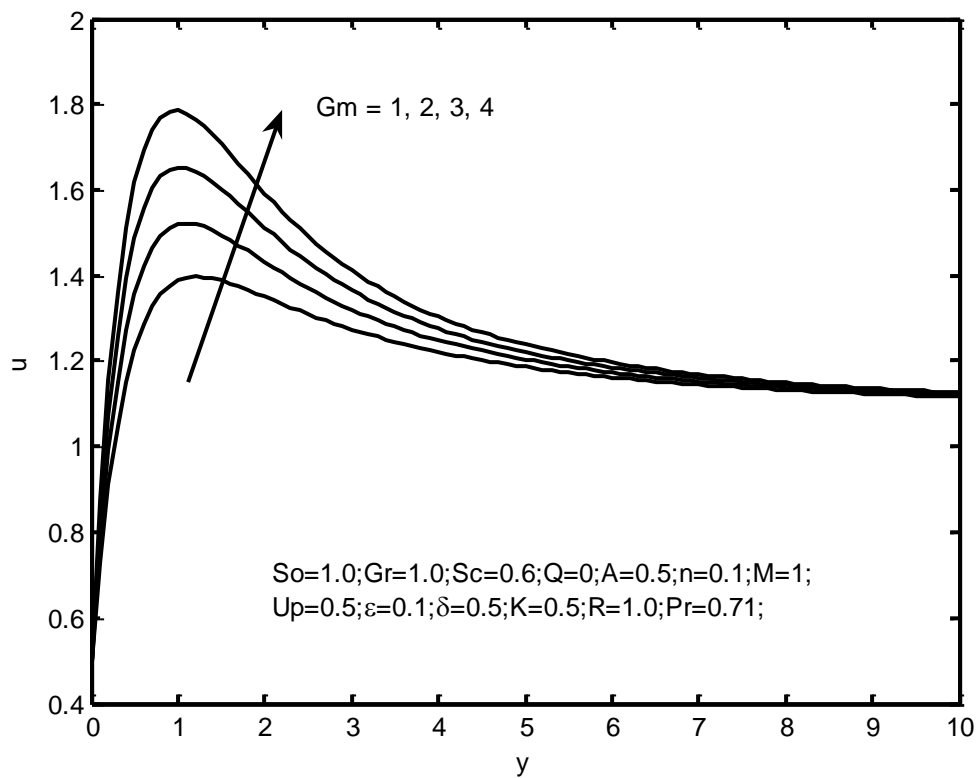
**Fig.4. Velocity profiles for different values of heat absorption parameter ( $Q$ ).**



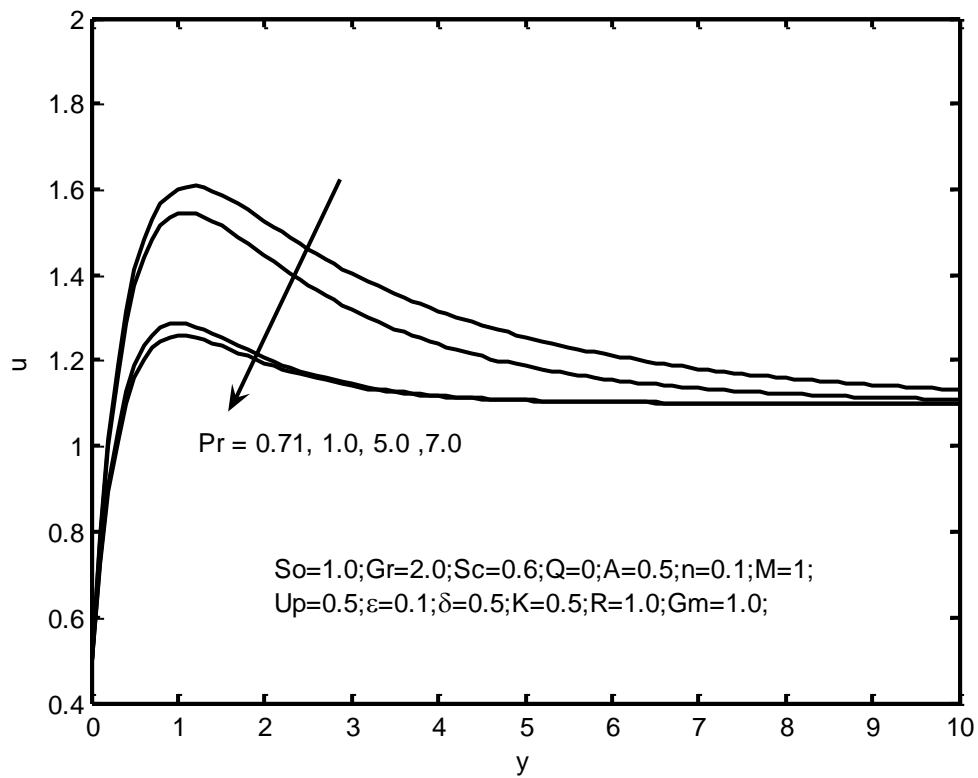
**Fig.5. Velocity profiles for different values of Soret number ( $So$ ).**



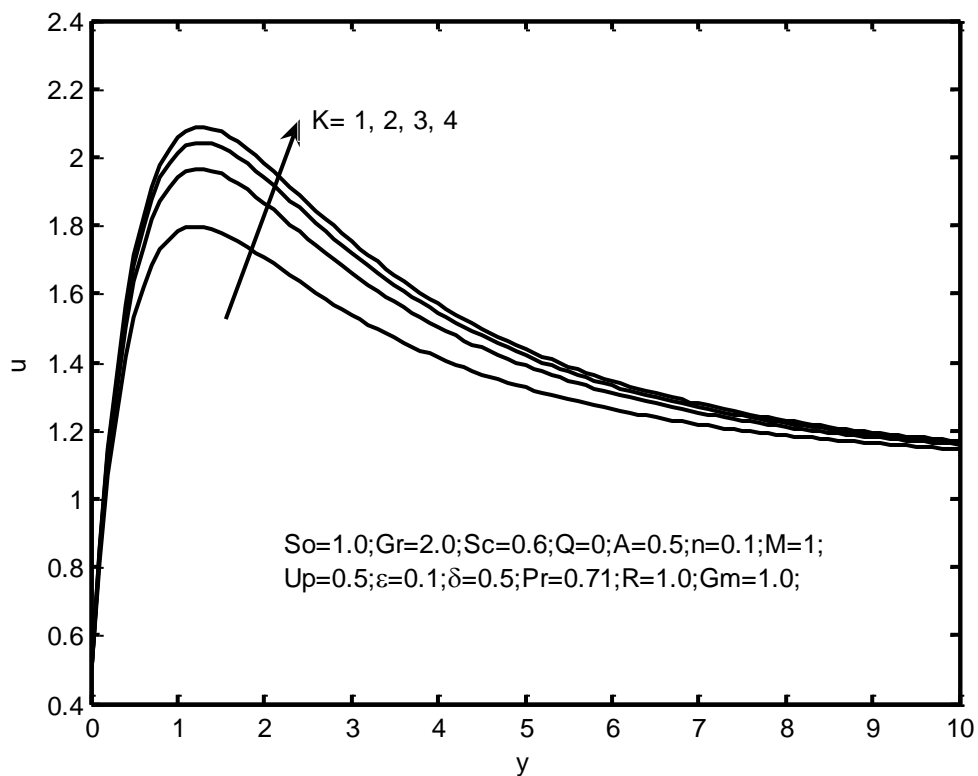
**Fig.6. Velocity profiles for different values of thermal Grashof number ( $Gr$ ).**



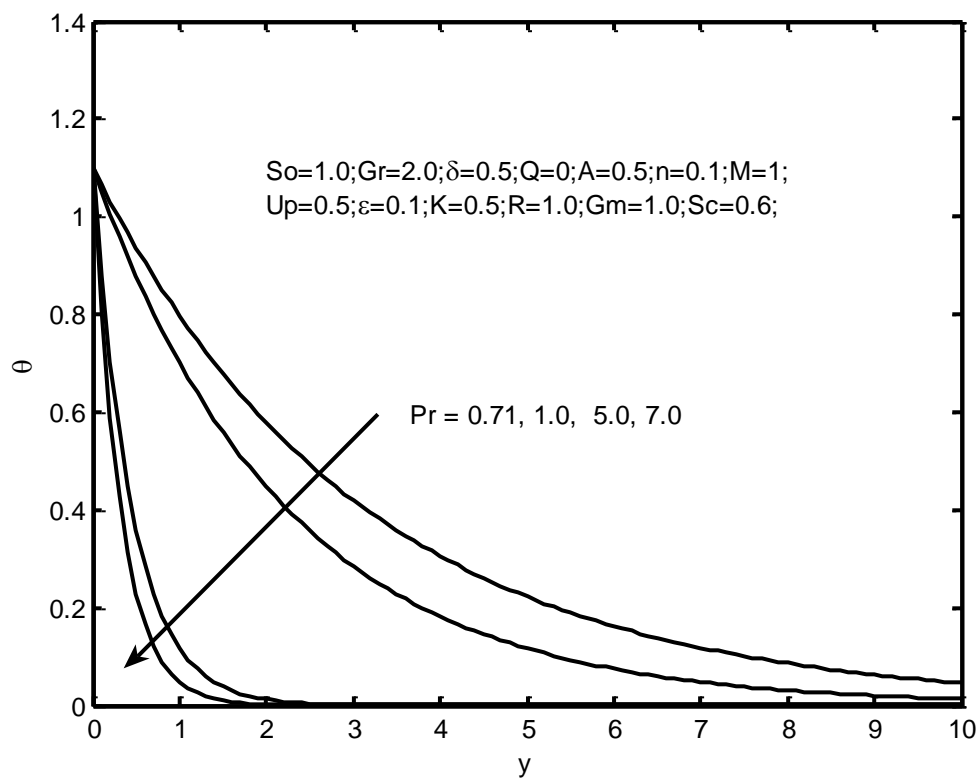
**Fig.7. Velocity profiles for different values of Solutal Grashof number ( $Gm$ ).**



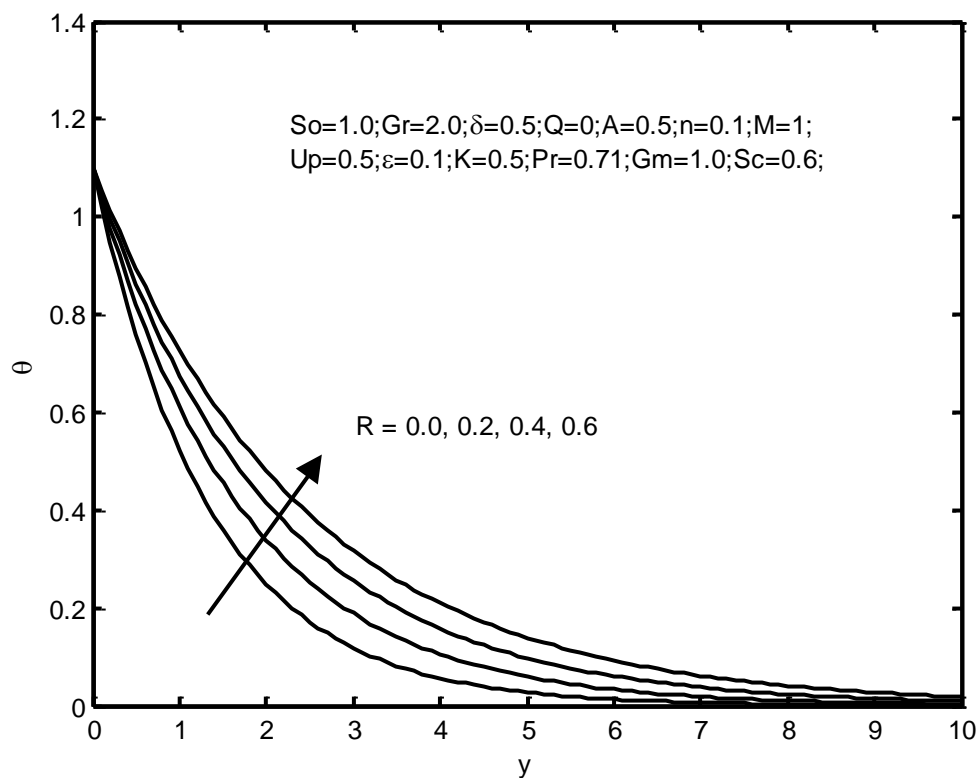
**Fig.8 Velocity profiles for different values of Prandtl number (Pr).**



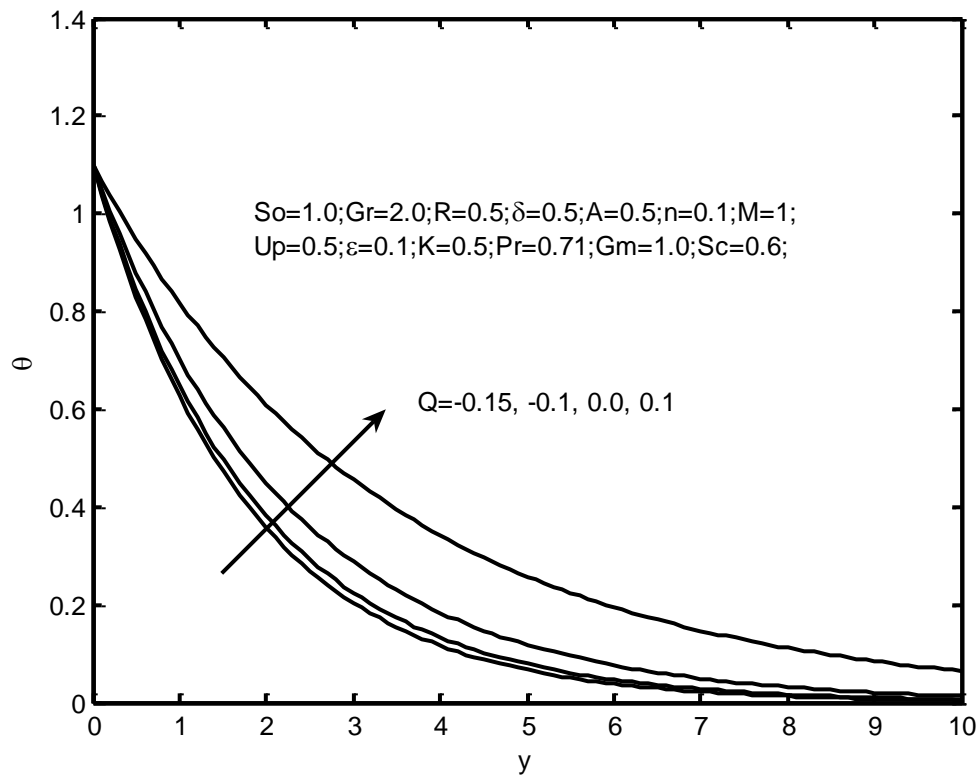
**Fig.9. Velocity profiles for different values of permeability parameter (K).**



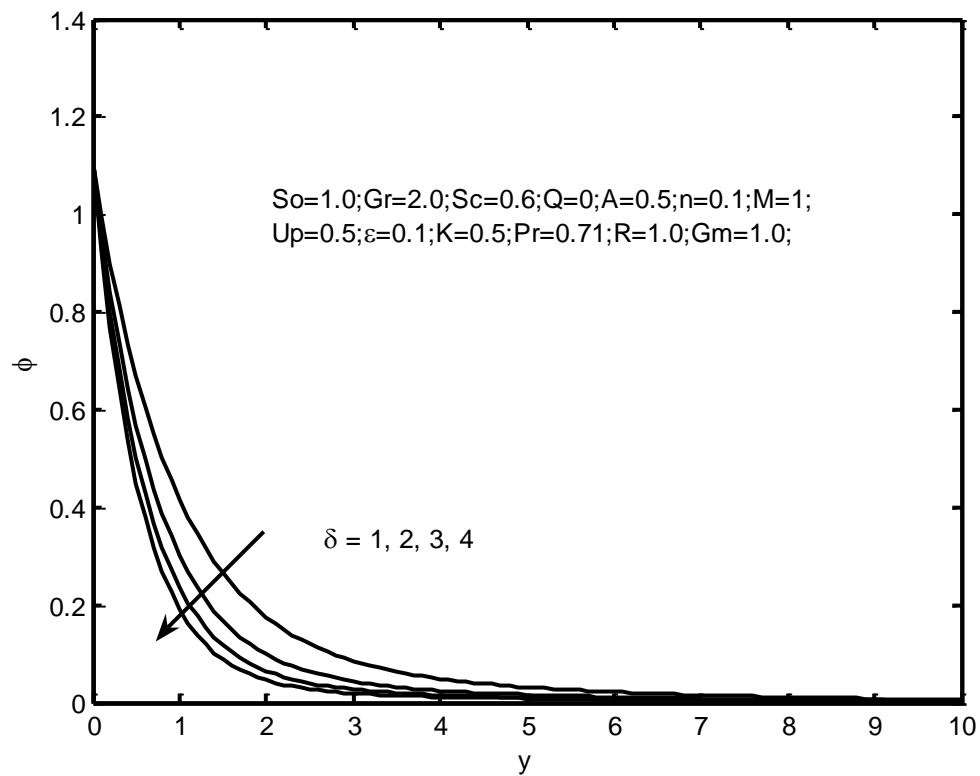
**Fig.10. Temperature profiles for different values of Prandtl number (Pr).**



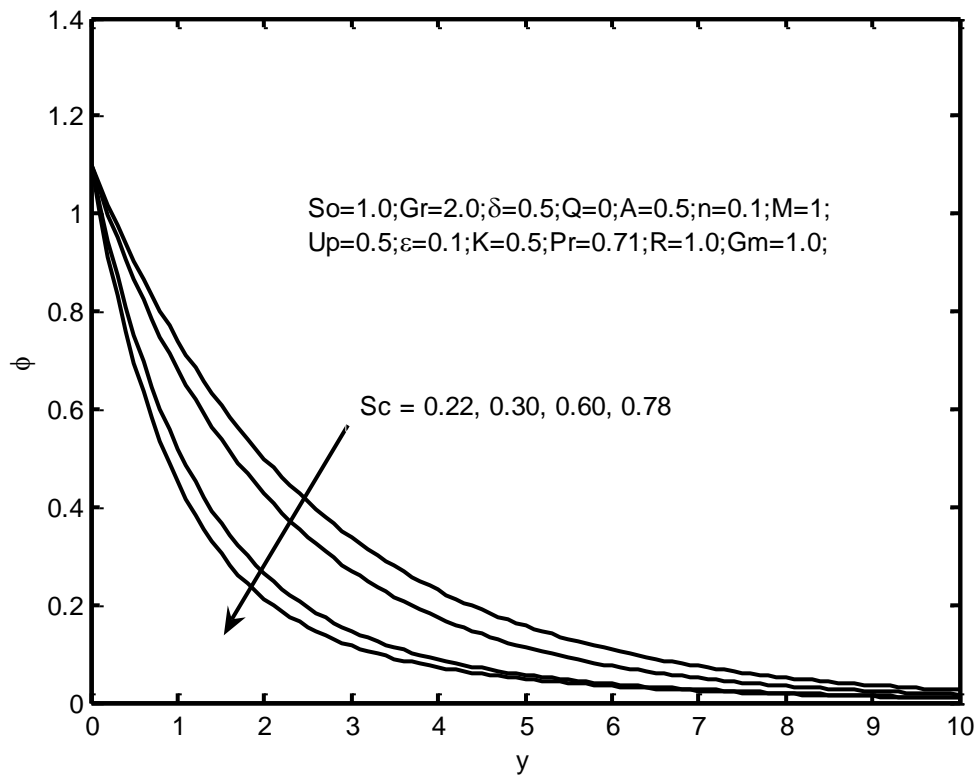
**Fig.11. Temperature profiles for different values of radiation parameter (R).**



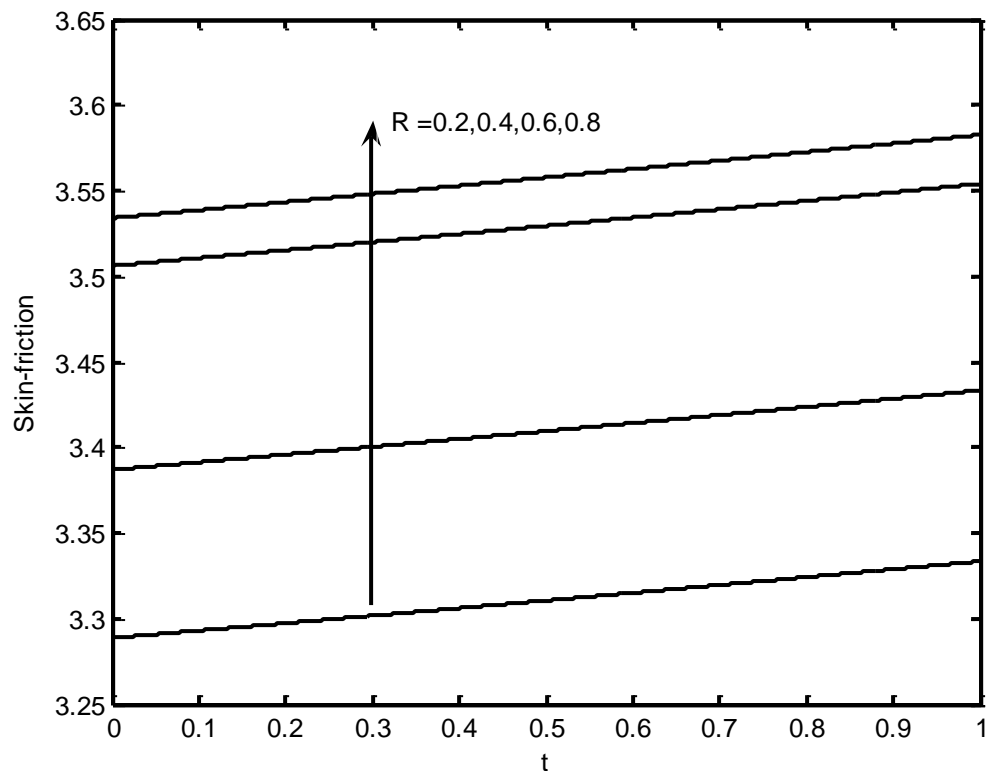
**Fig.12. Temperature profiles for different values of heat absorption parameter (Q)**



**Fig.13. Concentration profiles for different values of chemical reaction parameter ( $\delta$ ).**

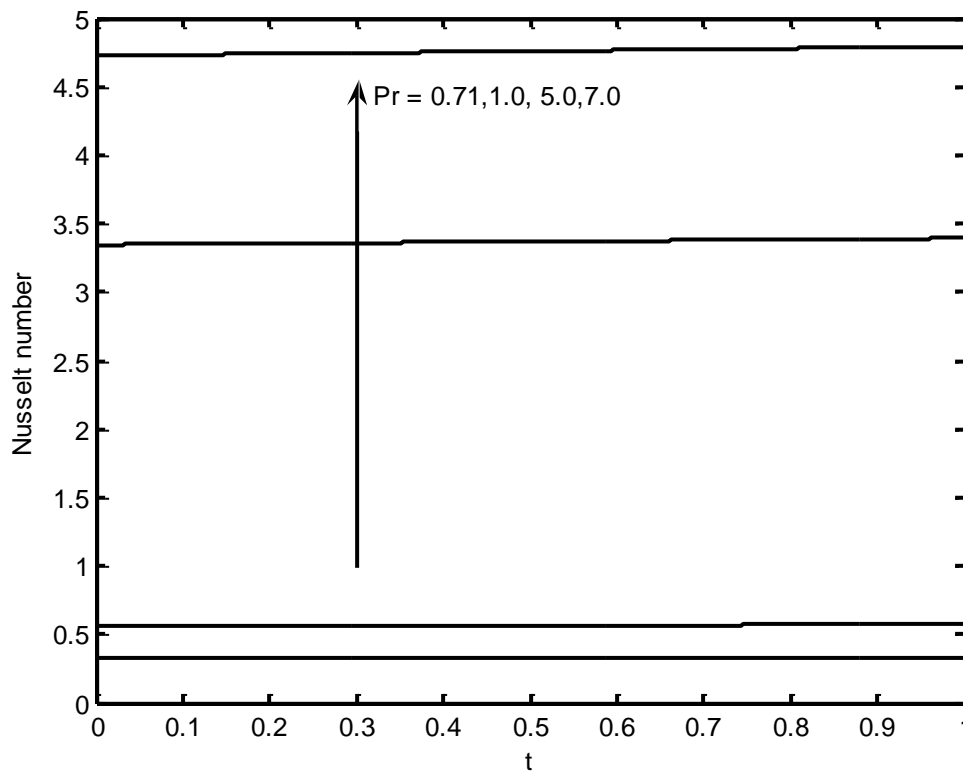


**Fig.14. Concentration profiles for different values of Schmidt number (Sc).**



**Fig.15. Skin-friction profiles for different values of radiation parameter.**





**Fig.16.Nusselt number profiles for different values of Nusselt numer.**

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**Appendix:**

$$F_1 = \frac{3Pr}{3+4R}, m_1 = \frac{F_1 + \sqrt{F_1^2 - 4F_1Q}}{2}, m_2 = \frac{F_1 + \sqrt{F_1^2 - 4F_1(Q-n)}}{2}, m_3 = \frac{Sc + \sqrt{Sc^2 + 4Sc\delta}}{2},$$

$$m_4 = \frac{Sc + \sqrt{Sc^2 + 4Sc(\delta+n)}}{2}, m_5 = \frac{1 + \sqrt{1+4N}}{2}, m_6 = \frac{1 + \sqrt{1+4(N+n)}}{2},$$

$$A_1 = AF_1m_1 / (m_1^2 - F_1m_1 + F_1(Q-n)), A_2 = 1 - A_1, A_3 = -SScm_1^2 / (m_1^2 - Scm_1 - \delta Sc),$$

$$A_4 = 1 - A_3, A_5 = -AScA_4, A_6 = -(AScA_4 + SScm_1^2A_1), A_7 = -SScm_2^2A_2,$$

$$A_8 = A_5 / [m_3^2 - Scm_3 - Sc(\delta+n)], A_9 = A_6 / [m_1^2 - Scm_1 - Sc(\delta+n)],$$

$$A_{10} = A_7 / [m_2^2 - Scm_2 - Sc(\delta+n)]; A_{11} = 1 - A_8 - A_9 - A_{10},$$

$$A_{12} = -(Gr + A_3Gm); A_{13} = -GmA_4, A_{14} = A_{12} / (m_1^2 - m_1 - N), A_{15} = A_{13} / (m_3^2 - m_3 - N),$$

$$A_{16} = P - A_{14} - A_{15} - 1, A_{17} = GrA_1 + GmA_9 - Am_1A_{14},$$

$$A_{18} = GrA_2 + GmA_{10}, A_{19} = GmA_8 + Am_3A_{15}, A_{20} = GmA_{11}, A_{21} = Am_5A_{16},$$

$$A_{22} = A_{17} / [(m_1^2 - m_1 - (N+n))], A_{23} = A_{18} / [m_2^2 - m_2 - (N+n)],$$

$$A_{24} = A_{19} / [m_3^2 - m_3 - (N+n)], A_{25} = A_{20} / [m_4^2 - m_4 - (N+n)],$$

$$A_{26} = A_{21} / [m_5^2 - m_5 - (N+n)], A_{27} = A_{22} + A_{23} + A_{24} + A_{25} - A_{26} - 1$$