DOMINATION IN SINGLE VALUED NEUTROSOPHIC GRAPHS

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Abstract

In a crisp graph *G=(V,E)*, we replace vertex and edge sets by single valued neutrosophic sets to generate the single valued neutrosophic graph *G=(A,B)*. In this paper, we define dominating set and dominating number of the single valued neutrosophic graph *G=(A,B)* also observe some bounds of the dominating number of *G=(A,B)*.

Keywords: Single valued neutrosophic graph, dominating set, domination number.

1. INTRODUCTION

In real word the most of the problems deals with incomplete, indeterminacy and in consistent information. Neutrosopic sets are the one of the powerful mathematical tool to deals these type of problems. Neutrosophic sets are introduced by Smarandache [6,7]. Neutrosophic sets are generalization of fuzzy sets, intuitionistic fuzzy sets and interval valued intuitionistic fuzzy sets. The single valued neutrosophic sets is a generalization of intuitionistic fuzzy . sets, in which three membership functions are independent and their value belong to [0,1].

Graph theory plays a major role in applied mathematics, domination is one of the major research feild in graph theory. In a crisp graph *G=(V,E)*, we replace vertex and edge sets by single valued neutrosophic sets to generate the single valued neutrosophic graph *G=(A,B)*. In this paper, we define dominating set and dominating number of the single valued neutrosophic graph *G=(A,B)* also observe some bounds of the dominating number of *G=(A,B)*.

2. PRELIMINARIES

Definition 2.1. Let X be a space of points (objects) with generic elements in X denoted by x, then the neutrosophic set A (NS A) is an object having the form

$$A = \{< x : T_A(x), I_A(x), F_A(x) >, x \in X\}$$

where the functions $T, I, F \to]^-0,1^+[$ define respectively the a truth-membership function, an indeterminacy-membership function, and a falsity-membership function of the element $x \in X$ to the set A with the condition

$$^-0 \le T_A(x) + I_A(x) + F_A(x) \le 3^+$$

The functions $T_A(x), I_A(x)$ and $F_A(x)$ are real standard or nonstandard subsets of $\left]^-0,1^+\right[$.

Definition 2.2. A single valued neutrosophic graphs *G=(A,B)* with vertex set V is defined by the set vertex and edge membership functions. The vertex membership functions

$$T_A : V \to [0,1]$$
$$I_A : V \to [0,1]$$
$$F_A : V \to [0,1]$$

T_A , denote the truth membership function , I_A, denote the indeterminacy membership function and F_A, denote the falsity membership function of the vertices $v_i \in V$ and $0 \le T_A(v_i) + I_A(v_i) + F_A(v_i) \le 3$ for all $v_i \in V$

The edge membership functions

$$T_B : E \subseteq V \times V \to [0,1]$$
$$I_B : E \subseteq V \times V \to [0,1]$$
$$F_B : E \subseteq V \times V \to [0,1]$$

are defined by

$$T_B(v_i v_j) \le \wedge \left[T_A(v_i), T_A(v_j)\right]$$
$$I_B(v_i v_j) \ge \vee \left[I_A(v_i), I_A(v_j)\right]$$
$$F_B(v_i v_j) \ge \vee \left[F_A(v_i), F_A(v_j)\right]$$

T_B , denote the truth membership function, I_B, denote the indeterminacy membership function and F_B, denote the falsity membership function of the edges $(v_i v_j) \in E$ and $0 \le T_B(v_i v_j) + I_B(v_i v_j) + F_B(v_i v_j) \le 3$ for all $(v_i v_j) \in E$.

Definition 2.3. In a single value neutrosophic graph $G = (A, B)$, any two vertices v_i *and* v_j are said to be adjacent, if

$$T_B(v_i v_j) = \wedge \left[T_A(v_i), T_A(v_j)\right]$$
$$I_B(v_i v_j) = \vee \left[I_A(v_i), I_A(v_j)\right]$$
$$F_B(v_i v_j) = \vee \left[F_A(v_i), F_A(v_j)\right]$$

The set of all adjacent vertices of a vertex v_i are neighbourhood set of the vertex v_i, and it is denoted by $N(v_i)$.

Definition 2.4. In a single value neutrosophic graph $G = (A, B)$, a vertices v_i is said to be isolated vertex, if there no edges adjacent on v_i .

Definition 2.5. In a single value neutrosophic graph $G=(A,B)$, a vertices v_i is said to be Pendent vertex, if there only one edges adjacent on v_i . A vertex in a single valued neutrosophic graph adjacent to the pendent vertex is called a support of the pendent edge.

Definition 2.6.A single value neutrosophic graph $G=(A,B)$ is said to be complete there is a strong edge between every pair of vertices.

$$T_B(v_iv_j) = \wedge\left[T_A(v_i), T_A(v_j)\right]$$
$$I_B(v_iv_j) = \vee\left[I_A(v_i), I_A(v_j)\right]$$
$$F_B(v_iv_j) = \vee\left[F_A(v_i), F_A(v_j)\right]$$

For every $v_i, v_j \in V$.

Definition 2.7. The cardinality of a vertex $v_i \in V$ in a single value neutrosophic graph $G=(A,B)$ is defined by $|v_i| = T_A(v_i) + I_A(v_i) + F_A(v_i)$

Definition 2.8. The neighbourhood T-degree of a vertex $v_i \in V$ in a single value neutrosophic graph $G=(A,B)$ is defined by $d_T(v_i) = \sum_{i \neq j} |v_j|$, for every vertex $v_j \in V$ such that v_iv_j is a T-adjacent vertices. (i.e)

$$T_B(v_iv_j) = \wedge\left[T_A(v_i), T_A(v_j)\right]$$
$$I_B(v_iv_j) \geq \vee\left[I_A(v_i), I_A(v_j)\right]$$
$$F_B(v_iv_j) \geq \vee\left[F_A(v_i), F_A(v_j)\right]$$

Definition 2.9. The neighbourhood I-degree of a vertex $v_i \in V$ in a single value neutrosophic graph $G=(A,B)$ is defined by $d_I(v_i) = \sum_{i \neq j} |v_j|$, for every vertex $v_j \in V$ such that v_iv_j is a I-adjacent vertices. (i.e)

$$T_B(v_iv_j) \leq \wedge\left[T_A(v_i), T_A(v_j)\right]$$
$$I_B(v_iv_j) = \vee\left[I_A(v_i), I_A(v_j)\right]$$
$$F_B(v_iv_j) \geq \vee\left[F_A(v_i), F_A(v_j)\right]$$

Definition 2.10. The neighbourhood F-degree of a vertex $v_i \in V$ in a single value neutrosophic graph $G=(A,B)$ is defined by $d_F(v_i)=\sum_{i\neq j}|v_j|$, for every vertex $v_j \in V$ such that v_iv_j is a F-adjacent vertices. (i.e)

$$\begin{aligned} T_B(v_iv_j) &\leq \wedge\left[T_A(v_i),T_A(v_j)\right] \\ I_B(v_iv_j) &\geq \vee\left[I_A(v_i),I_A(v_j)\right] \\ F_B(v_iv_j) &= \vee\left[F_A(v_i),F_A(v_j)\right] \end{aligned}$$

Definition 2.11. The neighbourhood degree of a vertex $v_i \in V$ in a single value neutrosophic graph $G=(A,B)$ is defined by $d_N(v_i)=\sum_{i\neq j}|v_j|$, for every vertex $v_j \in V$ such that

$$\begin{aligned} T_B(v_iv_j) &= \wedge\left[T_A(v_i),T_A(v_j)\right] \\ I_B(v_iv_j) &= \vee\left[I_A(v_i),I_A(v_j)\right] \\ F_B(v_iv_j) &= \vee\left[F_A(v_i),F_A(v_j)\right] \end{aligned}$$

Definition 2.12. The neighbourhood degree of a vertex $v_i \in V$ in a single value neutrosophic graph $G=(A,B)$ is defined by $td(v_i)=d_N(v_i)+|v_i|$

Definition 2.13. In a single value neutrosophic graph $G=(A,B)$ .If $v_i \in V$ is said to be a strong neighbour of vertex $v_j \in V$ such that

a) v_i *and* v_j are adjacent vertices in $G=(A,B)$

b) $d_N(v_i)\geq d_N(v_j)$

Definition 2.14. The order of the single value neutrosophic graph $G=(A,B)$ is defined by $O(G)=\sum_{i\in V}|v_i|$

Definition 2.15. The cardinality of an edge $v_iv_j \in E$ in a single value neutrosophic graph $G=(A,B)$ is defined by $|v_iv_j|=T_B(v_iv_j)+I_B(v_iv_j)+F_B(v_iv_j)$

Definition 2.16. The size of the single value neutrosophic graph $G=(A,B)$ is defined by

$$S(G)=\sum_{i,j\in V}|v_iv_j|$$

3. DOMINATION IN SNGLE VALUED NEUTROSOPHIC GRAPH

In this section we define dominating set and dominating number of SVNG and also observe some bounds and properties of domination number

Definition 3.1. In a single value neutrosophic graph $G=(A,B)$, if any two vertices v_i and v_j are said to be T-dominating each other such that there is a T-strong edge between the vertices v_i and v_j.

$$T_B(v_iv_j) = \wedge\left[T_A(v_i),T_A(v_j)\right]$$
$$I_B(v_iv_j) \geq \vee\left[I_A(v_i),I_A(v_j)\right]$$
$$F_B(v_iv_j) \geq \vee\left[F_A(v_i),F_A(v_j)\right]$$

Definition 3.2. In a single value neutrosophic graph $G=(A,B)$, the set $D\subseteq V$ is said to be a T-dominating set of $G=(A,B)$ such that every vertex $v\in V-D$ is T-dominated by at least one vertex $u\in D$.

Definition 3.3. In a single value neutrosophic graph $G=(A,B)$, the T-dominating set $D\subseteq V$ is said to be a minimal T-dominating set of $G=(A,B)$ such that $D-\{v\}$ is not a T-dominating set in $G=(A,B)$ for all $v\in D$.

Definition 3.4. In a single value neutrosophic graph $G=(A,B)$, the T-domination number is defined by the minimum cardinality among all the minimal T-dominating set of $G=(A,B)$. It is denoted by $\gamma_T(G)$.

Definition 3.5. In a single value neutrosophic graph $G=(A,B)$, if any two vertices v_i and v_j are said to be I-dominating each other such that there is a I-strong edge between the vertices v_i and v_j.

$$T_B(v_iv_j) \leq \wedge\left[T_A(v_i),T_A(v_j)\right]$$
$$I_B(v_iv_j) = \vee\left[I_A(v_i),I_A(v_j)\right]$$
$$F_B(v_iv_j) \geq \vee\left[F_A(v_i),F_A(v_j)\right]$$

Definition 3.6. In a single value neutrosophic graph $G=(A,B)$, the set $D\subseteq V$ is said to be a I-dominating set of $G=(A,B)$ such that every vertex $v\in V-D$ is I-dominated by at least one vertex $u\in D$.

Definition 3.7. In a single value neutrosophic graph $G=(A,B)$, the I-dominating set $D\subseteq V$ is said to be a minimal I-dominating set of $G=(A,B)$ such that $D-\{v\}$ is not a I-dominating set in $G=(A,B)$ for all $v\in D$.

Definition 3.8. In a single value neutrosophic graph $G=(A,B)$, the I-domination number is defined by the minimum cardinality among all the minimal I-dominating set of $G=(A,B)$. It is denoted by $\gamma_I(G)$.

Definition 3.9. In a single value neutrosophic graph $G=(A,B)$, if any two vertices v_i and v_j are said to be F-dominating each other such that there is a F-strong edge between the vertices v_i and v_j.

$$T_B(v_iv_j)\leq \wedge\left[T_A(v_i),T_A(v_j)\right]$$
$$I_B(v_iv_j)\geq \vee\left[I_A(v_i),I_A(v_j)\right]$$
$$F_B(v_iv_j)= \vee\left[F_A(v_i),F_A(v_j)\right]$$

Definition 3.10. In a single value neutrosophic graph $G=(A,B)$, the set $D\subseteq V$ is said to be a F-dominating set of $G=(A,B)$ such that every vertex $v\in V-D$ is F-dominated by at least one vertex $u\in D$.

Definition 3.11. In a single value neutrosophic graph $G=(A,B)$, the F-dominating set $D\subseteq V$ is said to be a minimal F-dominating set of $G=(A,B)$ such that $D-\{v\}$ is not a F-dominating set in $G=(A,B)$ for all $v\in D$.

Definition 3.12. In a single value neutrosophic graph $G=(A,B)$, the F-domination number is defined by the minimum cardinality among all the minimal F-dominating set of $G=(A,B)$. It is denoted by $\gamma_F(G)$.

Definition 3.13. In a single value neutrosophic graph $G=(A,B)$, if any two vertices v_i and v_j are said to be dominating each other such that there is a strong edge between the vertices v_i and v_j.

Definition 3.14. In a single value neutrosophic graph $G=(A,B)$, the set $D\subseteq V$ is said to be a dominating set of $G=(A,B)$ such that every vertex $v\in V-D$ is dominated by at least one vertex $u\in D$.

Definition 3.15. In a single value neutrosophic graph $G=(A,B)$, the dominating set $D \subseteq V$ is said to be a minimal dominating set of $G=(A,B)$ such that $D-\{v\}$ is not a dominating set in $G=(A,B)$ for all $v \in D$.

Definition 3.16. In a single value neutrosophic graph $G=(A,B)$, the domination number is defined by the minimum cardinality among all the minimal dominating set of $G=(A,B)$. It is denoted by $\gamma_{SVN}(G)$.

Example 3.1.

Consider the single valued neturosophic graph G=(A,B) . Let A be a single valued neturoscophic subset of V and let B single valued neturoscophic subset of E defined b

	A	b	c	d	e	f
T_A	.2	.2	.1	.4	.4	.3
I_A	.7	.2	.2	.6	.7	.4
F_A	.4	.7	.4	.2	.3	.3

	ab	ac	bc	bd	ce	de	ef
T_B	.2	.1	.1	.2	.1	.3	.3
I_B	.7	.7	.3	.6	.7	.8	.7
F_B	.7	.4	.7	.7	.5	.4	.3

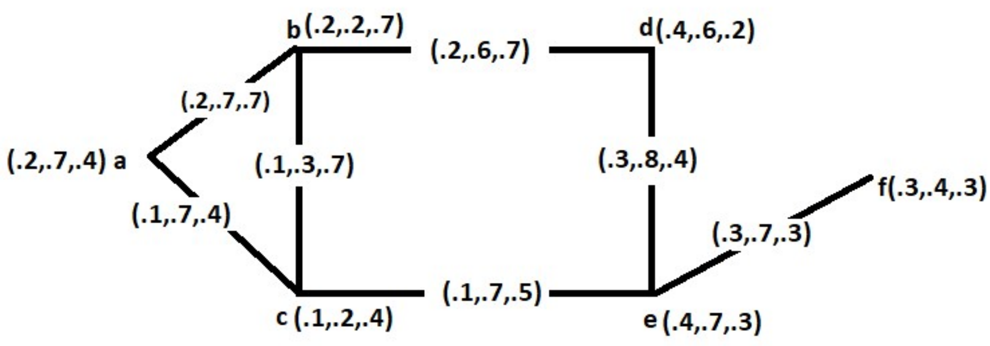

Figure 3.1, *G(A,B)*

In Figure 3.1,

- (*a* and *b*), (*b* and *d*) and (*e* and *f*) are dominated vertices
- (*a* and *b*), (*b* and *c*) , (*b* and *d*), (*c* and *e*) and (*e* and *f*) are T-dominated vertices
- (*a* and *b*), (*a* and *c*) , (*b* and *d*), (*c* and *e*) and (*e* and *f*) are I-dominated vertices
- (*a* and *b*), (*a* and *c*) , (*b* and *c*), (*b* and *d*) and (*e* and *f*) are F-dominated vertices
- The set $\{b,f\}$, $\{b,c,f\}$, $\{b,f\}$and $\{a,c,f\}$ is a T-dominating set, I-dominating set, F-dominating set and dominating set of G=(A,B) respectively.
- In G=(A,B) , $\gamma_T(G)=2.1$, $\gamma_I(G)=2.8$, $\gamma_F(G)=2.1$ and $\gamma_{SVN}(G)=3.0$

Theorem 3.1.

A single valued neutrosophic graph *G=(A,B)* with minimum dominating set $D \subseteq V$. Then the set $V - D$ is the dominating set of *G=(A,B)*.

Proof

Let $D - \{v\}$ be the minimum dominating set of *G = (A,B),*and if D is minimum dominating set then $D - \{v\}$is not an dominating set of *G = (A,B)*. Clearly every vertex $v_i \in V - D$ is adjacent to the vertex in $D - \{v\}$

$$T_B(v_i v_j) = \wedge \left[T_A(v_i), T_A(v_j)\right]$$
$$I_B(v_i v_j) = \vee \left[I_A(v_i), I_A(v_j)\right] \qquad \text{atleast a vertex } v_j \cup D$$
$$F_B(v_i v_j) = \vee \left[F_A(v_i), F_A(v_j)\right]$$

Clearly every vertex $v_j \in D$is adjacent to vertex in $v_i \in V - D$,therefore *V-D* is a dominating set of *G = (A,B)*.

Hence proved.

Theorem 3.2.

A single valued neutrosophic graph *G=(A,B)*then $\gamma_{SVN}(G) \leq 3\left[\frac{O(G)}{2}\right]$.

Proof

Let $D \subseteq V$ be the minimum dominating set of *G = (A,B)* and by theorem 1 $V - D$is a dominating set vertex having $\Delta_N(G) = d(v_i)$

$$\gamma_{SVN}(G) = Min\ (|D|, |V - D|)$$
$$\gamma_{SVN}(G) \leq \frac{O(G)}{2} \qquad \because V = D \cup (V - D)$$
$$\gamma_{SVN}(G) \leq \frac{3O(G^*)}{2} \qquad \because O(G) \leq 3\left[O(G^*)\right]$$

Hence proved.

Theorem 3.3.

Let D be a minimal dominating set of a single valued neutrosophic graph *G=(A,B)* if and only if every vertex $v \in D$ satisfies at least one of the following property.

1) There exists an edge $v \in V - D$ such that $N(v) \cap D = \{u\}$.
2) "u" is not dominated by the vertex in D.

Proof:

First we assume that D satisfies (i) or (ii) .

In (i), we note that u is the only vertex dominates v. In (ii), u is not dominated by the edges in D. Therefore we get $D-\{u\}$ is not a dominating set of G. This implies D is a minimal dominating set of G. Conversely, we assume that D is a minimal dominating set of G. Clearly $D-\{u\}$ is not a dominating set of G. This implies there exist a vertex $v \in V-(D-\{u\})$ which is not dominated by the vertex in $D-\{u\}$.

If $u=v$, we get u is adjacent to no vertex in D. If u is adjacent to vertices in D, obviously u is dominated by vertex in D. This implies $D-\{u\}$ is a dominating set of G. This contradict to our assumption D is minimal. Therefore e_1 is dominated by no vertices in D, property (ii) holds. If $u \neq v$, v is dominated by the vertices in D. Since D is a dominating set of G. The vertex v is not dominated by the vertices in $D-\{e_1\}$. Therefore we get $N(v) \cap D = \{u\}$. Hence proved

Theorem 3.4.

A single valued neutrosophic graph *G=(A,B)* with minimum dominating set $D \subseteq V$. Let $v_i \in V$ is the vertex having $\Delta_N(G) = d(v_i)$ then $\gamma_{SVN}(G) \leq O(G) - d(v_i)$

Proof

Let $D \subseteq V$ be the minimum dominating set of G = *(A,B)* and $v_i \in V$ be the vertex having $\Delta_N(G) = d(v_i)$. If $v_i \notin D$, there is a vertex the minimum $v_j \in D$ such that ,

$$T_B(v_i v_j) = \wedge \left[T_A(v_i), T_A(v_j)\right]$$
$$I_B(v_i v_j) = \vee \left[I_A(v_i), I_A(v_j)\right]$$
$$F_B(v_i v_j) = \vee \left[F_A(v_i), F_A(v_j)\right]$$

Therefore $V - N(v_i)$ is a dominating set of *G=(A,B)*

$$\gamma_{SVN}(G) \leq \left|V - N(v_i)\right|$$
$$\gamma_{SVN}(G) \leq O(G) - d(v_i)$$

Hence proved.

Conclusion

Neutrosophic sets is a generalization of the notion of fuzzy sets and intuitionistic fuzzy sets. Neutrosophic models gives more precisions, flexibility and compatibility to the system as compared to the classical, fuzzy and intuitionistic fuzzy models. In this paper, we have introduced dominating set , minimum dominating set and domination number future study, we plan to extend our research to various dominations parameters in single valued neutrosophic graphs and identify the bounds of the domination parameters.

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