On Neighbourhood Difference Cordial Labeling of Graphs

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Abstract— **In this paper we introduce a variation of difference cordial labeling of graphs called neighbourhood difference cordial labeling and we investigate the neighbourhood difference cordial labeling of path, cycle, complete graph, complete bipartite graph, the ladder graph, triangular ladder, diagonal ladder, the grid and the generalized Petersen graph.**

Keywords— **Path, cycle, complete graph, complete bipartite graph, star graph, wheel graph, ladder graph, triangular ladder, diagonal ladder, grid, generalized Petersen and neighbourhood difference cordial label.**

I. Introduction

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Graph labeling is the assignment of integers to the graph elements like vertices or edges or both subject to certain conditions. Rosa introduced the concept of labeling in 1967.

Graph labeling has a wide range of applications in Coding Theory, Radar location codes, cryptography, data mining, cloud computing, network security and Database Management system

Cahit introduced cordial labeling in 1987 and Ponraj etal introduced difference cordial labeling 2013. In 2015 Seoud and Salman studied difference cordial labeling of the ladder graph, triangular ladder, diagonal ladder, step ladder, two sided step ladder.

II. Preliminaries

Definition 1: A *graph* G is a finite non empty set of elements called vertices together with a set of pairs of distinct vertices of G which is called edges.

Definition 2: A *path* is a sequence of edges that begins at a vertex and travels from vertex to vertex along edges of the graph.

Definition 3: An *r regular* graph is a graph in which every vertex has degree r.

Definition 4: A *cycle* is the connected 2 regular graph of order n.

Definition 5: A *complete graph* on *n* vertices, denoted by is a simple graph that contains exactly one edge between each pair of distinct vertices.

Definition 6: A *complete bipartite graph* K_{r} is a graph that has its vertex set partitioned into two subsets of *m* and *n* vertices, respectively with an edge between two vertices if and only if one vertex is in the first subset and the other vertex is in the second subset.

Definition 7: A complete bipartite graph K is known as a *star graph.*

Definition 8: A *wheel* is a graph obtained from cycle by adding a new vertex and edges joining it to all vertices of cycle.

Definition 9: *Cartesian product* of $G \times$ of graphs and is a graph with vertex set $V(G) \times V($ and two vertices $(u,$ and $(u_1,$ in $G \times$ are adjacent if $u =$ and is adjacent to in or $v =$ and is adjacent to in .

Definition 10: Let denote the path on n vertices. For $m, n \geq 2, P_m \times$ is defined as the *two dimensional grid* with m rows and n columns and is denoted by M_m. Any vertex in i^{th} row and j^{th} column is denoted by .

Definition 11: A *ladder graph* is a planner undirected graph denoted by $L_n = P_n \times$ with vertices and $3n2$ edges.

Definition 12: *A triangular ladder* $TL_n, n \geq$ is a graph obtained from the ladder by adding the edges u_i, v_{i+1} for $1 \leq i \leq n -$ It has vertices and $4n$ - edges.

Definition 13: *Diagonal ladder* D is a ladder with additional edges $u_i v_i$ and u_{i+} with vertices and $5n$ - edges.

Definition 14: *The generalized Petersen graph* $P(n, k$ for $n >$ and $k >$ contains vertices $\{u_1, u_2, \ldots, u_n, v_1, v_2, v_3, \ldots, v$ and edges connecting $(u_i, u_{i+}$, $(u_i,$ and $(v_i, v_{i+}$, $i +$ and $i +$ are taken modulo *n*.

Definition 15: *Prism* is the cartesian product of with . Cylinder is the cartesian product of with . .

Definition 16: *Extended grid* is obtained by making each 4-cycle in M_m into a complete graph is called an extended grid. It is denoted by EX_m.
Definition 17: *The complement graph* (of a graph has $V($ as its vertex set, but two vertices of (are adjacent if and only if they are not adjacent in .
Definition 18: Let $G(p,$ be a graph. Let $f: V(G) \to \{1,2,3 \ldots$ be a function. For each assign the label $|f(u) - f($ f is called *difference cordial labeling* if f is one to one and $|e_f(1) - e_f(0)| \leq$ where $e_f($and $e_f($denotes the number of edges labeled with 1 and labeled not with 1 respectively. Any graph with difference cordial labeling is called difference cordial graph.

Neighbourhood difference cordial labeling :

Let $G = (V, E)$be a graph where V and E are the vertex and edge sets of G. . Let $f: V \to \{1, 2, 3, \ldots, p\}$ be a function. For each edge uv assign the label $|f(u) - f(v)|$. f is called a neighborhood difference cordial labeling if f is one to one map and for every vertex $v \in V(G)$, $|ef_v(0) - ef_v(1)| \leq 1$, where $ef_v(1)$and $ef_v(0)$ denote the number of edges incident with v and labeled with 1 and not labeled with 1 respectively. A graph with neighborhood difference cordial labeling is called neighbourhood difference cordial graph.

The neighbourhood difference cordial labeling of path, cycle, wheel, complete graph, complete bipartite graph, the ladder graph, triangular ladder, star, the grid and the Petersen graph are investigated.

Theorem 1: Anypath $P_n$is neighbourhood difference cordial graph.

Proof:

Let $V = \{v_1, v_2, v_{3}, \ldots, v_n\}$ be the vertex set of P_n

Define $f: V \to \{1, 2, 3, \ldots, n\}$, as follows

$f(v_1) = 1, f(v_{2i}) = 2i + 1, f(v_{2i+1}) = 2i,\ 1 \leq i \leq \frac{n-1}{2}$,when n is odd.

$f(v_{2i}) = 2i - 1, f(v_{2i-1}) = 2i,\ 1 \leq i \leq \frac{n}{2}$, when n is even. Obviously at any vertexv_i

$|ef_{v_i}(1) - ef_{v_i}(0)| = 0$. Hence $P_n$is neighborhood difference cordial graph.

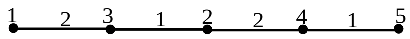


Fig 1: Path

Lemma 1: For any vertex $v \in V(G) ef_v(1) \leq 2$.

Proof: Suppose there exists a vertex $v \in V(G)$ such that $ef_v(1) > 2$, which is not possible since for any number there exists at most one successor and one predecessor. Therefore for every vertex $v \in V(G)$, $ef_v(1) \leq 2$

Theorem 2: Any even cycleC_{2n} is neighbourhood difference cordial graph.

Proof:

Let$V = \{v_1, v_2, \ldots, v_{2n}\}$

Define $f: V \to \{1, 2, 3, \ldots, 2n\}$

$f(v_{2i}) = 2i - 1, f(v_{2i-1}) = 2i, 1 \leq i \leq n$

It is very clear $|ef\, v_i(1) - ef\, v_i(0)| = 0$.

Hence C_{2n} is neighbourhood difference cordial graph.

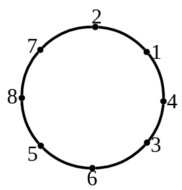


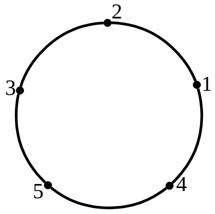
Fig 2: Even Cycle

Theorem 3: Any odd cycle C_n is not neighbourhood difference cordial graph.

Proof:

Let vertex set of C_n be $\{v_0, v_1, v_2, \ldots, v_{n-1}\}$. If suppose the edge $v_0 v_1$ is labeled as 1 , $v_1 v_2$as 0, $v_2 v_3$ as 1and so on . Since it has odd number of edges the edge $v_{n-1} v_0$ is labeled as 1. So at v_0

$|ef_{v_0}(0) - ef_{v_0}(1)| = 2 > 1$ whic is a contradiction . Hence any odd cycle is not neighbourhood cordial graph.



$Fig\ 3: oddcycle$

Remark 2: Any odd Cycle is difference cordial graph but not neighbourhood cordial graph.

Theorem 4: Anygraph $G(p,q)$ with $\delta > 3$ is not neighborhood difference cordial graph.

Since $\delta > 3$, at the vertex v with label 1, $ef_v(1) = 1$ and $ef_v(0) > 2$,

$|ef_v(0) - ef_v(1)| > 1$ .Hence G is not a neighbourhood difference cordial graph.

It is easy to prove the following results.

Corollary 1: Any r regular graph is not neighbourhood difference cordial graph for $r \geq 4$

Corollary 2: Anycomplete bipartite graph $K_{m,n}$, $m \geq 4$, $n \geq 4$ is not neighbourhood difference cordial graph.

Remark 3: In any neighbourhood cordial graph G there exist atleast two vertices $u, v \in V(G)$ with $\deg(u) \leq 3$ and $\deg(v) \leq 3$.

Suppose $\deg(u) > 3$ and $\deg(v) > 3$, label the vertices u ,v with 1 and p respectively, $ef_u(1) = 1$, $ef_u(0) > 2$ and $|ef_u(1) - ef_u(0)| > 1$, similarly $|ef_v(1) - ef_v(0)| > 1$. HenceG is not neighbourhood cordial graph, a contradiction.

Theorem 5: Anygraph with $\Delta > 5$ is not neighbourhood difference cordial graph.

Proof:

Suppose the vertex $v \in V(G)$ with $\deg(v) > 5$, by lemma 1, $ef_v(1) \leq 2$, then $ef_v(0) > 3$.$|ef_v(0) - ef_v(1)| > 1$. Hence the graph is not neighbourhood difference cordial graph.

Theorem 6: If $G(p,q)$ is a neighbourhood difference cordial graph then

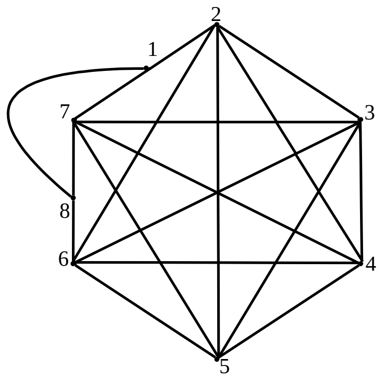
$$q \leq \frac{1}{2}(5p - 4)$$

Proof: Let $G(p,q)$ be a graph

Suppose the graph has $p - 2$ vertices of degree ≤ 5 and two vertices of degree ≤ 3 ,then

$$2q \leq 5(p-2) + 2(3) = 5p - 4$$

Hence $q \leq \frac{1}{2}(5p - 4)$.



$Fig\ 4: Illustration\ for\ q = \frac{1}{2}(5p-4)$

Corollary 3: Any star graph is not neighbourhood difference dordial graph when the degree of the centre vertex is greater than 5.

Corollary 4: Any wheel graph is not neighbourhood difference dordial graph when the degree of the centre vertex is greater than 5.

Theorem 7: AnyLadder L_n is neighbourhood difference cordial graph.

Proof:

Let$V = \{v_1, v_2, \dots, v_{2n}\}$

Define $f: V \to \{1, 2, 3, \dots, 2n\}$

$f(v_{ij}) = j + (i-1)n$,$1 \le j \le n$,$1 \le i \le 2$

Ladder has 4 corner vertices of degree 2 and $2n-4$ vertices of degree 3.

It has vertical edges and horizontal edges.By the abovelabeling the horizontal edges are labeled with 1 and vertical edges are not labeled with 1. Corner vertex v has one vertical and one horizontal edge.

Case 1:

At v_{11} , $|f(v_{11}) - f(v_{12})| = |1-2| = 1.$

$|f(v_{11}) - f(v_{21})| = |1-(n+1)| = n.$

$ef_{v_{11}}(1) = 1$and $ef_{v_{11}}(0) = 1 \therefore \left|ef_{v_{11}}(1) - ef_{v_{11}}(0)\right| = 0$

Case 2:

At v_{1n} , $\left|f(v_{1n}) - f(v_{1,n-1})\right| = |n-(n-1)| = 1$

$|f(v_{1n}) - f(v_{2n})| = |n-(n+n)| = n$

$ef_{v_{1n}}(1) = 1$ and $ef_{v_{1n}}(0) = 1 \therefore \left|ef_{v_{11}}(1) - ef_{v_{11}}(0)\right| = 0$

Case 3:

At v_{21} , $|f(v_{21}) - f(v_{22})| = |1-2| = 1.$

$|f(v_{21}) - f(v_{11})| = |n+1-1| = n.$

$ef_{v_{21}}(1) = 1$and $ef_{v_{21}}(0) = 1 \therefore \left|ef_{v_{21}}(1) - ef_{v_{21}}(0)\right| = 0$

Case 4:

At v_{2n} , $\left|f(v_{2n}) - f(v_{2(n-1)})\right| = |(n+n)-(n+n-1)| = 1.$

$|f(v_{2n}) - f(v_{1n})| = |(n+n)-n)| = |n| = n.$

$ef_{v_{21}}(1) = 1$and $ef_{v_{21}}(0) = 1 \therefore \left|ef_{v_{21}}(1) - ef_{v_{21}}(0)\right| = 0.$

Case 5:

At any vertex v_{1j} ,$2 \le j \le n-1.$

$\left|f(v_{1j}) - f(v_{1j+1})\right| = |j-(j+1)| = |j-j-1| = 1$

$\left|f(v_{1j}) - f(v_{1j-1})\right| = |j-(j-1)| = |j-j+1| = 1.$

$\left|f(v_{1j}) - f(v_{2j})\right| = |j-(j+n)| = |j-j-n| = n.$

$ef_{v_{1j}}(0) = 1$ and $ef_{v_{1j}}(1) = 2$, $2 \le j \le n-1.$

$\therefore \left|ef_{v_{1j}}(0) - ef_{v_{1j}}(1)\right| = 1$, $2 \le j \le n-1.$

Case 6:

At any vertex v_{2j} , $2 \le j \le n-1$

$\left|f(v_{2j}) - f(v_{2(j+1)})\right| = |j+n-(j+1+n)| = 1.$

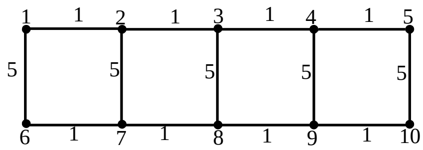
$\left|f(v_{2j}) - f(v_{2(j-1)})\right| = |j+n-(j-1+n)| = 1.$

$\left|f(v_{2j}) - f(v_{1j})\right| = |j+n-j| = |j-j-n| = n.$

$ef_{v_{2j}}(0) = 1$ and $ef_{v_{2j}}(1) = 2, 2 \le j \le n-1.$

$\therefore \left|ef_{v_{1j}}(0) - ef_{v_{1j}}(1)\right| = 1$, $2 \le j \le n-1,$

So L_n is neighborhood difference cordial graph.



$fig\ 5: Ladder$

Theorem 8: AnydiagonalLadder DL_n is neighbourhood difference cordial graph.

Proof:

Let$V = \{v_1, v_2, \dots, v_{2n}\}$

Define $f: V \to \{1, 2, 3, \dots, 2n\}$

$f(v_{ij}) = j + (i-1)n, 1 \leq j \leq n, 1 \leq i \leq 2$

The diagonal ladder has 4 corner vertices of degree 3 and 2n-4 vertices of degree 5.

Case 1:

At v_{11}, $ef_{v_{11}}(1) = 1$ and $ef_{v_{11}}(0) = 2 \therefore |ef_{v_{11}}(1) - ef_{v_{11}}(0)| = 1.$

Case 2:

At v_{1n}, $|f(v_{1n}) - f(v_{1(n-1)})| = |n - (n-1)| = 1$

$|f(v_{1n}) - f(v_{2(n-1)})| = |n - (n+n-1)| = n-1.$

$|f(v_{1n}) - f(v_{2n})| = |n - (n+n-1)| = n-1.$

$ef_{v_{1n}}(1) = 1$and $ef_{v_{1n}}(0) = 2 \therefore |ef_{v_{1n}}(1) - ef_{v_{1n}}(0)| \leq 1.$

Case 3:

At v_{21}, $|f(v_{21}) - f(v_{22})| = |1-2| = |-1| = 1.$

$|f(v_{21}) - f(v_{11})| = |n+1-1| = n.$

$|f(v_{21}) - f(v_{12})| = |1+n-2| = n-1$

$ef_{v_{21}}(1) = 1$ and $ef_{v_{21}}(0) = 2 \therefore |ef_{v_{21}}(1) - ef_{v_{21}}(0)| \leq 1.$

Case 4:

At v_{2n}, $|f(v_{2n}) - f(v_{2(n-1)})| = |2n - (2n-1) + j| = 1.$

$f(v_{2n}) - f(v_{1n}) = |n+n-n| = n.$

$f(v_{2n}) - f(v_{1(n-1)}) = |n+n-(n-1)| = n+1.$

$ef_{v_{2n}}(1) = 1$ and $ef_{v_{2n}}(0) = 1 \therefore |ef_{v_{2n}}(1) - ef_{v_{2n}}(0)| = 0 \leq 1.$

At any vertex v_{1j}, $2 \leq j \leq n-1$

$|f(v_{1j}) - f(v_{1(j+1)})| = |j - (j+1)| = 1.$

$|f(v_{1j}) - f(v_{1(j-1)})| = |j - (j-1)| = 1.$

$|f(v_{1j}) - f(v_{2j})| = |j - (j+n)| = n.$

$|f(v_{1j}) - f(v_{2(j+1)})| = |j - (j+1+n)| = n+1.$

$|f(v_{1j}) - f(v_{2(j-1)})| = |j - (j-1+n)| = n-1.$

$ef_{v_{1j}}(1) = 2$, $ef_{v_{1j}}(0) = 3$, $2 \leq j \leq n-1.$

$\therefore |ef_{v_{1j}}(1) - ef_{v_{1j}}(0)| = 1, 2 \leq j \leq n-1$

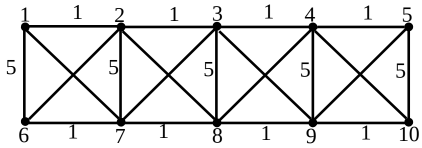
At any vertex v_{2j}, $2 \leq j \leq n-1.$

$f(v_{2j}) = j + n, 1 \leq j \leq n$

$\left|f(v_{2j}) - f\left(v_{2(j+1)}\right)\right| = |j + n - (j + n + 1)| = 1.$
$\left|f(v_{2j}) - f\left(v_{2(j-1)}\right)\right| = |j + n - (j + n - 1)| = 1.$
$\left|f(v_{2j}) - f(v_{1j})\right| = |j + n - j| = |j - j - n| = n.$
$\left|f(v_{2j}) - f\left(v_{1(j+1)}\right)\right| = |j + n - (j + 1)| = n - 1.$
$\left|f(v_{2j}) - f\left(v_{1(j-1)}\right)\right| = |j + n - (j - 1)| = n + 1.$
$ef_{v_{2j}}(1) = 2 \, , ef_{v_{2j}}(0) = 3, 2 \le j \le n - 1.$

$\therefore \left|ef_{v_{2j}}(1) - ef_{v_{2j}}(0)\right| = 1, 2 \le j \le n - 1.$

Hence any DL_n is neighbourhood difference cordial graph.



$fig\ 6: Diagonal\ ladder$

Corollary 7: Extended grid is not neighbourhood cordial graph since $\Delta = 8$

Theorem 9: AnytriangularLadder TL_n is neighbourhood difference cordial graph.
Let$V = \{v_1, \ v_2 \ , \dots \ , \ v_{2n}\}$
Define $f: V \to \{1, 2, 3, \dots, 2n\}$
$f(v_{ij}) = f(v_{ij}) = j + (i - 1)n, 1 \le j \le n \, , 1 \le i \le 2$
The triangular ladder has 2 corner vertices of degree 3, 2 corner vertices of degree 2 and 2n-4 vertices of degree 4.

Case 1:
At v_{11} , $ef_{v_{11}}(1) = 1$ and $ef_{v_{11}}(0) = 2 \therefore \left|ef_{v_{11}}(1) - ef_{v_{11}}(0)\right| = 1.$

Case 2:
At v_{1n} , $ef_{v_{1n}}(1) = 1$ and $ef_{v_{1n}}(0) = 1 \therefore \left|ef_{v_{1n}}(1) - ef_{v_{1n}}(0)\right| = 0.$

Case 3:
At v_{21} , $ef_{v_{21}}(1) = 1$ and $ef_{v_{21}}(0) = 1 \therefore \left|ef_{v_{21}}(1) - ef_{v_{21}}(0)\right| = 0.$

Case 4:
At v_{2n} , $\left|f(v_{2n}) - f\left(v_{2(n-1)}\right)\right| = |(n + n) - (n + n - 1)| = 1.$
$|f(v_{2n}) - f(v_{1n})| = |(n + n - n)| = n.$
$\left|f(v_{2n}) - f\left(v_{1(n-1)}\right)\right| = |(n + n - (n - 1)| = n + 1.$
$ef_{v_{21}}(1) = 1$ and $ef_{v_{21}}(0) = 2 \therefore \left|ef_{v_{21}}(1) - ef_{v_{21}}(0)\right| = 1.$

Case 5:
At any vertex v_{1j} ,$2 \le j \le n - 1.$
$\left|f(v_{1j}) - f(v_{1j+1})\right| = |j - (j + 1)| = 1.$
$\left|f(v_{1j}) - f\left(v_{1(j-1)}\right)\right| = |j - (j - 1)| = 1.$
$\left|f(v_{1j}) - f(v_{2j})\right| = |j - (j + n)| = n.$
$\left|f(v_{1j}) - f\left(v_{2(j+1)}\right)\right| = |j - (j + 1 + n)| = n + 1.$
$ef_{v_{1j}}(1) = 2 \, , ef_{v_{1j}}(0) = 2 \, , 2 \le j \le n - 1.$

$\therefore \left|ef_{v_{1j}}(1) - ef_{v_{1j}}(0)\right| = 0 \, , 2 \le j \le n - 1.$

Case 6:

At any vertex v_{2j} , $2 \leq j \leq n-1$.

$|f(v_{2j}) - f(v_{2(j+1)})| = |j+n-(j+n+1)| = 1.$

$|f(v_{2j}) - f(v_{2(j-1)})| = |j+n-(j+n-1)| = 1.$

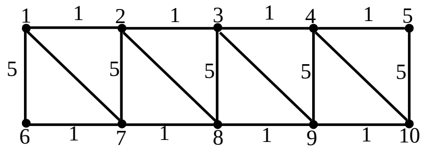
$|f(v_{2j}) - f(v_{1j})| = |j+n-j| = |j-j-n| = n.$

$|f(v_{2j}) - f(v_{1(j-1)})| = |j+n-(j-1)| = n-1.$

$ef_{v_{2j}}(1) = 2 , ef_{v_{2j}}(0) = 2 , 2 \leq j \leq n-1.$

$\therefore \left|ef_{v_{2j}}(1) - ef_{v_{2j}}(0)\right| = 0 , 2 \leq j \leq n-1.$

So TL_n is neighbourhood difference cordial graph.



$fig\ 7: Triangular\ ladder$

Theorem 10: Anyprism $C_n \times P_2$ is neighbourhood difference cordial graph.

Proof:

Let $V = \{v_1, v_2, \dots, v_{2n}\}$

Define $f: V \rightarrow \{1, 2, 3, \dots, 2n\}$

$f(v_{ij}) = j + (i-1)n , 1 \leq j \leq n , 1 \leq i \leq 2$

At any vertex v_{1j} ,$2 \leq j \leq n-1$.

$|f(v_{1j}) - f(v_{1(j+1)})| = |j-(j+1)| = |j-j-1| = 1.$

$|f(v_{1j}) - f(v_{1(j-1)})| = |j-(j-1)| = |j-j+1| = 1.$

$|f(v_{1j}) - f(v_{2j})| = |j-(j+n)| = |j-j-n| = n.$

$ef_{v_{1j}}(1) = 2 , ef_{v_{1j}}(0) = 1, 1 \leq j \leq n$

$$\therefore \left|ef_{v_{1j}}(1) - ef_{v_{1j}}(0)\right| = 1, 1 \leq j \leq n$$

At any vertex v_{2j} , $2 \leq j \leq n-1$

$f(v_{2j}) = j+n, 1 \leq j \leq n$

$|f(v_{2j}) - f(v_{2(j+1)})| = |j+n-(j+n+1)| = 1$

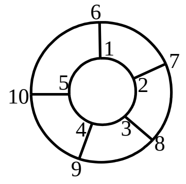
$|f(v_{2j}) - f(v_{2(j-1)})| = |j+n-(j+n-1)| = 1$

$|f(v_{2j}) - f(v_{1j})| = |j+n-j| = |j-j-n| = n$

$ef_{v_{2j}}(1) = 2 , ef_{v_{2j}}(0) = 1 , 1 \leq j \leq n$

$\therefore \left|ef_{v_{2j}}(1) - ef_{v_{2j}}(0)\right| = 1 , 1 \leq j \leq n$

So $C_n \times P_2$ is neighbourhood difference cordial graph.



$Fig\ 8: C_5 \times P_2$

Theorem 11: Anycylinder $C_n \times P_m$ is neighbourhood difference cordial graph.

Let$V = \{v_1, v_2, \dots, v_{mn}\}$

Define $f: V \to \{1, 2, 3, \dots, n, n+1, \dots, 2n, \dots mn\}$

$f(v_{1j}) = j, 1 \le j \le n$

$f(v_{ij}) = (i-1)(n+1) + j, 1 \le j \le n-(i-1), 2 \le i \le m$

$f(v_{ij}) = (i-1)n + (i-1)(mod\ m) + j, 1 \le j \le n-(i-1), 2 \le i \le m$ $f(v_{i(n-(i-2))}) = (i-1)n + 1$

$f\left(v_{(i(n-(i-3))mod\ n)}\right) = (i-1)n + 2$

$f\left(v_{(i(n-(i-4))mod\ n)}\right) = (i-1)n + 3$

....

$f(v_{(i(n-(i-m))\ mod\ n)}) = (i-1)n + (m-1)$

$f(v_{in}) = (i-1)(n+1)$

Cylinder consists of $2n$ vertices of degree 3 and $mn - 2n$ vertices of degree 4.

Case 1:

At $v_{1j}, 1 \le j \le n$

$|f(v_{1j}) - f(v_{1(j+1)})| = |j - (j+1)| = 1$

$|f(v_{1j}) - f(v_{1(j-1)})| = |j - (j-1)| = 1$

$|f(v_{1j}) - f(v_{2j})| = |j - (j+n)| = n$

$ef_{v_{1j}}(1) = 2, ef_{v_{1j}}(0) = 1, 1 \le j \le n$

$\left| ef_{v_{ij}}(1) - ef_{v_{ij}}(0) \right| = 1, 1 \le j \le n$

Case 2:

At $v_{nj}, 2 \le j \le n-1$

$|f(v_{nj}) - f(v_{n(j+1)})| = |j - (j+1)| = 1$

$|f(v_{nj}) - f(v_{n(j-1)})| = |j - (j-1)| = 1$

$|f(v_{nj}) - f(v_{(n+1)j})| = |j + (n-1)n - (j + (n)n)| = n$

$ef_{v_{nj}}(1) = 2, ef_{v_{nj}}(0) = 1, 1 \le j \le n$

$\left| ef_{v_{nj}}(1) - ef_{v_{nj}}(0) \right| = 1, 1 \le j \le n$

Case 3:

At any vertex $v_{ij}, 2 \le j \le n-1, 2 \le i \le m-1$

$|f(v_{ij}) - f(v_{i(j+1)})| = |j + (i-1)n - (j + 1 + (i-1)n)| = 1$

$|f(v_{ij}) - f(v_{i(j-1)})| = |j + (i-1)n - (j + (i-1)n - 1)| = 1$

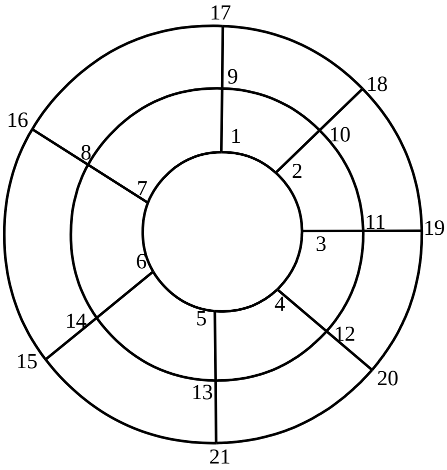
$|f(v_{ij}) - f(v_{i+1j})| = |j + (i-1)n - (j + in)| = n$

$|f(v_{ij}) - f(v_{i-1j})| = |j + (i-1)n - (j + (i-2)n)| = n$

$ef_{v_{ij}}(1) = 2, ef_{v_{ij}}(0) = 2, 1 \le j \le n, 1 \le i \le m$

$\left| ef_{v_{ij}}(1) - ef_{v_{ij}}(0) \right| = 0, 1 \le j \le n, 1 \le i \le m$

So $C_n \times P_m$ is neighbourhood difference cordial graph.



$Fig\ 9: Cylinder\ \ C_5\ \times p_3$

Theorem 12: Generalised petersen graph is a neighbourhood difference cordial graph.

Let $V = \{v_1,\ v_2\ , \dots\ ,\ v_{2n}\}$

Define $f: V \rightarrow \{1, 2, 3, \dots, 2n\}$

The petersen graph has $2n$ vertices and $3n$ edges. Label the vertices in the outer layer as $u_1, u_2\ , \dots\ , u_{n-1}\ , u_n$ and the vertices in the inner layer as $v_1, v_2, \dots\ , v_{n-1}, v_n$.

Case 1: When *n* is not divisible by k

$f(u_i) = i, 1\ \leq i \leq n.$

$f\big(v_{(k(i-1)+1) mod\ n}\big) = n + (i-1) + 1, 1 \leq i \leq n$

At any $v_i\ ,\ 1 \leq i \leq n$

$\big|f\big(v_{(k(i-1)+1) mod\ n}\big) - f\big(v_{(ki+1)\ mod\ n}\big)\big| = \ |(n + (i-1) + 1) - (n + i + 1)| = 1.$

$\big|f\big(v_{(k(i-1)+1) mod\ n}\big) - f\big(v_{(k(i-2)+1) mod\ n}\big)\big|$

$= \ |(n + (i-1) + 1) - (n + (i-2) + 1)| = 1$

$\big|f\big(v_{(k(i-1)+1) mod\ n}\big) - f\big(u_{(k(i-1)+1) mod\ n}\big)\big|$

$= \ |(n + (i-1) + 1) - (k(i-1) + 1) mod\ n|$

$= |(n + i) - (k(i-1) + 1) mod\ n|$

$\big|ef_{v_i}(1) - ef_{v_i}(0)\big| = |2 - 1| = 1\ , 2 \leq i \leq n.$

At any $u_i\ ,\ 2 \leq i \leq n$

$|f(u_i) - f(u_{i+1})| = |i - (i+1)| = 1$

$|f(u_i) - f(u_{i-1})| = |i - (i-1)| = 1$

$\big|f\big(v_{(k(i-1)+1) mod\ n}\big) - f\big(u_{(k(i-1)+1) mod\ n}\big)\big|$

$= \ |(n + (i-1) + 1) - (k(i-1) + 1) mod\ n|$

$= |(n + i) - (k(i-1) + 1) mod\ n|$

$\big|ef_{u_i}(1) - ef_{u_i}(0)\big| = |2 - 1| = 1$

At u_1,

$|f(u_1) - f(u_2)| = |1 - 2| = 1$

$|f(u_1) - f(u_n)| = |1 - n| = n - 1$

$|f(v_1) - f(u_1)| = |n + 1 - 1| = n$

$\big|ef_{u_1}(1) - ef_{u_1}(0)\big| = |1 - 2| = 1$

Case 2: when n is divisible by k.

$f(u_i) = i, 1 \le i \le n.$

$f(v_1) = n + 1$

$f(v_{(k(i-1)+j) mod\ n}) = n + 1 + (i - 1) + (j - 1)\frac{n}{k}, 1 \le i \le \frac{n}{k}, 1 \le j \le k$

At any v_i, $2 \le i \le \frac{n}{k}$

$|f(v_{(k(i-1)+j) mod\ n}) - f(v_{(ki+j) mod\ n})|$

$= \left|\left(\left(n + 1 + (i - 1) + (j - 1)\frac{n}{k}\right) - \left(n + 1 + i + (j - 1)\frac{n}{k}\right)\right)\right| = 1$

$|f(v_{(ki+j) mod\ n}) - f(v_{(k(i+1)+j) mod\ n})|$

$= \left|\left(\left(n + 1 + i + (j - 1)\frac{n}{k}\right) - \left(n + 1 + (i + 1) + (j - 1)\frac{n}{k}\right)\right)\right| = 1$

$|f(v_{(k(i-1)+j) mod\ n}) - f(u_{(k(i-1)+j) mod\ n})|$

$= \left|\left(n + 1 + (i - 1) + (j - 1)\frac{n}{k}\right) - (k(i - 1) + j)\right| \ne 1$

$|f(v_{(ik+(n-1)) mod n}) - f(v(_{((i+1)k+(n-1)) mod\ n})| = \left|\left(n + (n - 1)\left\lceil\frac{n}{k}\right\rceil + 1 + i\right) - \right.$
$\left.(n + (n - 1)\left\lceil\frac{n}{k}\right\rceil + 1 + i + 1))\right| = 1$

$|f(v_{(ik+(n-1)) mod n}) - f(u_i)| = \left|(n + (n - 1)\left\lceil\frac{n}{k}\right\rceil + 1 + i) - i\right| = n + (n - 1)\left\lceil\frac{n}{k}\right\rceil + 1$

$|ef_{v_i}(1) - ef_{v_i}(0)| = 1, 2 \le i \le n.$

At v_1

$|f(v_1) - f(v_2)| = 1$

$|f(v_1) - f(v_n)| = |(n + 1) - (2n)| = n - 1$

$|f(v_1) - f(v_{(ik+(n-1)) mod\ n})| = \left|(n + 1) - (n + 1 + (n - 1)\left\lceil\frac{n}{2}\right\rceil)\right| = (n - 1)\left\lceil\frac{n}{2}\right\rceil$

$|f(v_1) - f(u_1)| = |n + 1 - 1| = n$

$|ef_{v_1}(1) - ef_{v_1}(0)| = 1.$

At any u_i, $2 \le i \le n - 1$,

$|f(u_i) - f(u_{i+1})| = |i - (i + 1)| = 1$

$|f(u_i) - f(u_{i-1})| = |i - (i - 1)| = 1$

$|f(v_{(k(i-1)+j) mod\ n}) - f(u_{(k(i-1)+j) mod\ n})|$

$= \left|\left(n + 1 + (i - 1) + (j - 1)\frac{n}{k}\right) - (k(i - 1) + j)\right| \ne 1$

$|ef_{u_i}(1) - ef_{u_i}(0)| = 1, 2 \le i \le n - 1$

At u_1,

$|f(u_1) - f(u_2)| = 1$

$|f(u_1) - f(u_n)| = |1 - n| = n - 1$

$|f(v_1) - f(u_1)| = |n + 1 - 1| = n$

$|ef_{u_1}(1) - ef_{u_1}(0)| = 1$

Hence generalised petersen graph is neighbourhood difference cordial graph.

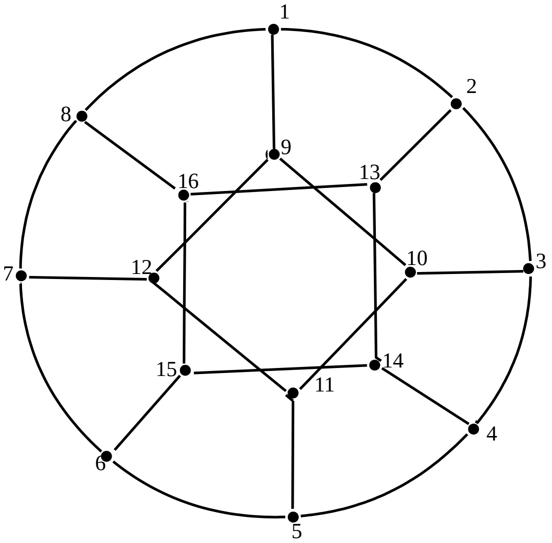


Fig 10: Petersen graph P(8,2)

Theorem 13: Complement of a neighborhood difference cordial graph $G(p,q)$ is not neighborhood difference cordial graph if $p > 9$.

Proof:

Let u be a vertex whose degree is the minimum. By theorem 5 , $\deg(u) \leq 3$. In G^C $\deg(u) \geq p-4$ Suppose $P > 9$, then $\deg(u) > 5$, then G^C is not neighbourhood difference cordial graph

Conclusion: A new labeling called neighbourhood difference cordial labeling is defined and the neighborhood difference cordial labeling of path, cycle, wheel, complete graph, complete bipartite graph, ladder graph, triangular ladder, star, grid and generalized petersen graph are investigated.

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