AN APPLICATION TO FUZZY PROGRAMMING APPROACH TO MULTI-OBJECTIVE TRANSPORTATION PROBLEM

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ABSTRACT

The multi-objective transportation problem (MOTP) is a particular type of vector minimum problem in which objectives conflict with each other and constraints are all equality type. This paper proposes an algorithm for solving bi-objective transportation problem (BOTP) through fuzzy programming approach. A mathematical model has been developed to discuss about BOTP that is characterized by non-linear (exponential and hyperbolic) membership function. For the application of proposed approach, fish export situation in Tamil Nadu is studied and a problem is solved.

Keywords: Transportation Problem, bi-objective transportation problem (BOTP), fuzzy nonlinear programming, LINGO, multi-objective transportation problem (MOTP).

1. INTRODUCTION

The problem of minimizing the total cost/time/distance of transportation has been studied since long and is well known. Transportation problem is basically developed by Hitchcock (1941) and stepping stone method was developed by Charnes and Cooper (1954). Tang and Wang (1996) proposed a type of fuzzy nonlinear programming problems using a genetic algorithmic approach along the weighted gradient direction with mutation. They found a group of solutions with acceptable membership degrees. Verma et al. (1997) studied a special type of vector-minimum linear programming problems using a special type of non-linear (hyperbolic and exponential) membership functions to solve the multi-objective transportation problem. They considered constraints of equality type; all the given objectives are non-commensurable and conflict with each other.

Gupta and Mehlawat (2009) studied a pair of fuzzy primal–dual linear programming problems using an exponential membership function. Kagade and Bajaj (2009) studied a special class of vector minimum linear programming problems to the multi-objective assignment problem and solved using linear and nonlinear membership functions.

Lohgaonkar and Bajaj (2010) studied fuzzy programming with linear, exponential and hyperbolic membership function for solving multi-objective capacitated transportation problem.

De and Yadav (2011) formulated a mathematical model for multi-criteria transportation problem using exponential membership function. Zangiabadi and Maleki (2013) presented an application of fuzzy goal programming considering linear membership function to solve multi-objective transportation problem using non-linear (hyperbolic and exponential) membership functions. Singh and Yadav (2015) developed an approach for solving intuitionistic fuzzy non-linear programming problem by transforming the intuitionistic fuzzy model into a fuzzy model using the concept of component wise optimization. Also, they proposed a non-linear membership function to convert the fuzzy model into a crisp model. Christi and Kalpana (2016) considered fuzzy interval numbers to solve multi-objective transportation problem using non-linear membership functions.

Rest of the paper is organized as follows: Following the introduction, model representation of BOTP is given in Section 2 that is followed by an algorithm. In section 3, we formalized a bi-objective transportation problem and the model is solved using LINGO 17.0 software package. In Section 4, we have given the comparison between proposed and ideal solution. Finally, Section 5 includes conclusion.

2. Bi-Objective Transportation Problem (BOTP)

In real life, the transportation problem involves multiple, have no common measure and conflicting objective functions. Similar to classic multi-objective transportation problem, an item is to be transported from m sources to n destinations. The capacities are $a_1, a_2, \dots, a_m$ and $b_1, b_2, \dots, b_n$, assuming $a_i, b_j \geq 0$ and must satisfy $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$ (equilibrium condition). The penalty C_{ij}^{α} associated with transporting a unit of an item from source i to destination j for the α^{th}- criterion. Also, Z_α and C_{ij}^{α} are related to α^{th} penalty criterion. Here C_{ij}^{α} may be cost, distance, delivery time, quantity of goods received or etc. A variable x_{ij} represents the unknown quantity to be transported from source i to destination j.

$$min\, Z_\alpha = \sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij}^{\alpha} x_{ij}\,, \quad \alpha = 1, 2.$$

Such that

$$\sum_{j=1}^{n} x_{ij} = a_i, \quad i = 1,2, \dots, m\,,$$

$$\sum_{i=1}^{m} x_{ij} = b_j\,, \quad j = 1,2, \dots, n$$

where $0 \leq x_{ij}\ \forall\, i\ and\ j\,, C_{ij} \geq 0$.

2.1 Proposed Algorithm for solving bi-objective transportation problem.

For solving BOTP, the simplest method given here is summarized in the following steps.

Step 1. Find the average value for each cell(i, j).

Step 2. For *step 1*, construct a new table having membership value.

Step 3. Solve *step 2* by Stepping stone method and find Z_1.

Step 4. Find the product of each cell (i, j).

Step 5. Solve *step 4* by Stepping stone method and find Z_2.

2.2 Algorithm for solving BOTP using Fuzzy programming technique

To solve BOTP based on the fuzzy programming technique one can use the following steps.

Step 1. Solve the BOTP by using the proposed algorithm.

Step 2. According to each solution, one can find the pay-off matrix for each objective as follows.

	$Z_1(x)$	$Z_2(x)$
$x^{(1)}$	Z_{11}	Z_{12}
$x^{(2)}$	Z_{21}	Z_{22}

where $x^{(1)}, x^{(2)}$ are the isolated optimal solutions of the two different TP's for two different objective functions, $Z_{ij} = Z_j(x^i), (i = 1,2 ; j = 1,2)$ be the i^{th} row and j^{th} column element of the pay-off matrix.

Step 3. For each objective find the best (L_α) and the worst (U_α) corresponding to the set of solutions, where $U_\alpha = (Z_\alpha)_{max}, L_\alpha = (Z_\alpha)_{min}, \alpha = 1,2$ for the α^{th} objective function Z_α.

Step 4. Define a membership functions μ_α (linear-μ_α^l, hyperbolic-μ_α^h, exponential- μ_α^e) for the α^{th} objective.

- A linear membership function is defined as follows.

$$\mu^l(Z_\alpha) = \begin{cases} 1, & if \quad Z_\alpha \le L_\alpha \\ 1 - \frac{Z_\alpha - L_\alpha}{U_\alpha - L_\alpha}, & if \ L_\alpha < Z_\alpha < U_\alpha \\ 0, & if \quad Z_\alpha \ge U_\alpha \end{cases} \qquad (1)$$

If we use the linear membership function as defined in (1), then an equivalent crisp model can be formulated as:

$$maximize: \eta,$$

Subject to.

$$\eta \le \mu(Z_\alpha)$$

$$\sum_{j=1}^{n} x_{ij} = a_i, \ \ i = 1,2, \dots, m$$

$$\sum_{i=1}^{m} x_{ij} = b_j , \ \ j = 1,2, \dots , n$$
$$0 \leq x_{ij} \ \forall \, i \ \ and \, j \, , \ \eta \geq 0 \, .$$

- An exponential membership function is defined by

$$\mu^e(Z_\alpha) = \begin{cases} 1, & if \ \ Z_\alpha \leq L_\alpha \\ \frac{e^{-s\phi_\alpha(x)} - e^{-s}}{1-e^{-s}} , & if \ L_\alpha < Z_\alpha < U_\alpha \\ 0 \, , & if \ \ Z_\alpha \geq U_\alpha \end{cases} \quad (2)$$

where $\phi_\alpha(x) = \frac{Z_\alpha - L_\alpha}{U_\alpha - L_\alpha}$, $\alpha = 1,2$ and s is a non-zero parameter.

If we use the exponential membership function as defined in(2), then an equivalent crisp model for the fuzzy model can be formulated as follows:

$$maximize{:}\, \eta$$

Subject to.

$$\eta \leq \frac{e^{-s\phi_\alpha(x)} - e^{-s}}{1 - e^{-s}} \, , \alpha = 1,2$$
$$\sum_{j=1}^{n} x_{ij} = a_i , \ \ i = 1,2, \dots , m$$
$$\sum_{i=1}^{m} x_{ij} = b_j , \ \ j = 1,2, \dots , n$$
$$0 \leq x_{ij} \ \forall \, i \ \ and \, j \, , \ \eta \geq 0 \, .$$

- A hyperbolic membership function is defined by

$$\mu^h(Z_\alpha) = \begin{cases} 1, & if \ \ Z_\alpha \leq L_\alpha \\ \frac{1}{2} \frac{e^{[\frac{U_\alpha + L_\alpha}{2} - Z_\alpha(x)]\delta_\alpha} - e^{-[\frac{U_\alpha + L_\alpha}{2} - Z_\alpha(x)]\delta_\alpha}}{e^{[\frac{U_\alpha + L_\alpha}{2} - Z_\alpha(x)]\delta_\alpha} + e^{-[\frac{U_\alpha + L_\alpha}{2} - Z_\alpha(x)]\delta_\alpha}} + 0.5, & if \ L_\alpha < Z_\alpha < U_\alpha \\ 0 \, , & if \ \ Z_\alpha \geq U_\alpha \end{cases} \quad (3)$$

where $\delta_\alpha = \frac{6}{U_\alpha - L_\alpha}$

If we use the hyperbolic membership function as defined in (3), then the equivalent crisp model for the fuzzy model can be formulated as follows:

$$maximize \, : \, x_{m \times n+1}$$

Subject to.

$$\delta_k Z_k(x) + x_{m \times n+1} \leq \frac{\delta_k}{2} (U_k + L_k), k = 1,2$$

$$\sum_{j=1}^{n} x_{ij} = a_i, \; i = 1,2,\dots,m$$

$$\sum_{i=1}^{m} x_{ij} = b_j, \; j = 1,2,\dots,n$$

$$0 \leq x_{ij} \; \forall \, i \text{ and } j, \eta \geq 0, x_{m\times n+1} \geq 0$$

where $x_{m\times n+1} = tanh^{-1}(2\eta - 1)$.

Step 5. Solve the equivalent crisp model defined in Step 4, and solve using LINGO software. The solution obtained will be the optimal solution of BOTP model.

3. MODEL IMPLEMENTATION

3.1. Fish Export in transportation case study and data description

The suppliers export fishes from different sources to different destinations. A survey was conducted in and around Nagapattinam district, Tamil Nadu, India and the data was collected by conducting questionnaire from whole sellers. Three different suppliers are taken in this study namely from Nagapattinam (S_1), Karaikal(S_2) and Vedharanyam(S_3) and they export to three major places namely Tuticorin(D_1), Ernakulam(D_2), and Chennai(D_3) . The problem constructed here is to minimize total time and distance taken to export fishes.

The upper left corner in each cell depicts the time (table 1) and distance (table 2) of transportation on the corresponding route and lower right corner in each cell depicts the supply in tonnes and the minimum requirement on that route. Dealer is unable to meet whole demand of customer during the planned period. However, dealer has to fulfill demand of all customers within certain time. That is the reliability of demand of the each customer from the suppliers has to be met and to minimize the total transportation time and total transportation distance.

In order to demonstrate the problem discussed above, the time and distance of transporting fishes from i^{th} source to j^{th} destination is given in the following table 1 and table 2.

Destinations → Sources ↓	D_1	D_2	D_3	Supply (in Tonnes)
S_1	8	13	7	12
S_2	9	15	6	7
S_3	10	14	9	8
Minimum requirement (in Tonnes)	7	11	9	42

Table 1. Data for Time

Destinations → Sources ↓	D_1	D_2	D_3	Supply (in Tonnes)
S_1	320	448	350	12
S_2	367	505	247	7
S_3	371	510	366	8
Minimum requirement (in Tonnes)	7	11	9	42

Table 2. Data for distance

3.2 Problem Formulation

The problem is formulated as:

$$minimize\ Z_1 = 8x_{11} + 13x_{12} + 7x_{13} + 9x_{21} + 15x_{22} + 6x_{23} + 10x_{31} + 14x_{32} + 9x_{33}$$

$$minimize\ Z_2 = 320x_{11} + 448x_{12} + 350x_{13} + 367x_{21} + 505x_{22} + 247x_{23} + 371x_{31} + 510x_{32} + 366x_{33}$$

Subject to

$$\sum_{i=1}^{3} x_{i1} = 7,\ \sum_{i=1}^{3} x_{i2} = 11,\ \sum_{i=1}^{3} x_{i3} = 8,\ \sum_{j=1}^{3} x_{1j} = 12,\ \sum_{j=1}^{3} x_{2j} = 7,\ \sum_{j=1}^{3} x_{3j} = 8$$

where $0 \leq x_{ij},\ i = 1,2,3$ and $j = 1,2,3$.

Step 1. By solving the BOTP by using the proposed algorithm we find the optimal solution as

$$x^{(1)}: x_{11} = 3, x_{12} = 0, x_{13} = 9,\ x_{21} = 0,\ x_{22} = 7,\ x_{23} = 0, x_{31} = 4, x_{32} = 4, x_{33} = 0$$

$$x^{(2)}: x_{11} = 1, x_{12} = 11, x_{13} = 0,\ x_{21} = 0,\ x_{22} = 0,\ x_{23} = 7, x_{31} = 6, x_{32} = 0, x_{33} = 2$$

Step 2. According to each solution, we can find the pay-off matrix for each objective as follows.

	$Z_1(x)$	$Z_2(x)$
$x^{(1)}$	288	11169
$x^{(2)}$	271	9935

Step 3. The best and the worst bounds of each objective function can be written as follows:

$$U_1 = max(288,271) = 288\ ;\ U_2 = max(11169,9935) = 11169$$

$$L_1 = min(288,271) = 271\ ;\ U_2 = min(11169,9935) = 9935$$

Step 4. Using a linear membership function as defined (1), the crisp model can be presented as follows:

$$maximize\ \eta$$

Subject to

$8x_{11} + 13x_{12} + 7x_{13} + 9x_{21} + 15x_{22} + 6x_{23} + 10x_{31} + 14x_{32} + 9x_{33} + \eta \leq 288$

$320x_{11} + 448x_{12} + 350x_{13} + 367x_{21} + 505x_{22} + 247x_{23} + 371x_{31} + 510x_{32} + 366x_{33} + \eta \leq 11169$

$$\sum_{i=1}^{3} x_{i1} = 7, \quad \sum_{i=1}^{3} x_{i2} = 11, \quad \sum_{i=1}^{3} x_{i3} = 8, \quad \sum_{j=1}^{3} x_{1j} = 12, \sum_{j=1}^{3} x_{2j} = 7, \quad \sum_{j=1}^{3} x_{3j} = 8$$

where $0 \leq x_{ij}$, $i = 1,2,3$; $j = 1,2,3$ and $\eta \geq 0$.

Now, using an exponential membership function as defined (2), the crisp model can be presented as follows:

$maximize: \eta$

$$e^{\frac{(-8x_{11}-13x_{12}-7x_{13}-9x_{21}-15x_{22}-6x_{23}-10x_{31}-14x_{32}-9x_{33}+271)}{17}} - (1 - e^{-1})\eta \geq e^{-1}$$

$$e^{\frac{(-320x_{11}-448x_{12}-350x_{13}-367x_{21}-505x_{22}-247x_{23}-371x_{31}-510x_{32}-366x_{33})}{1234}} - (1 - e^{-1})\eta \geq -e^{-1}$$

$$\sum_{i=1}^{3} x_{i1} = 7, \quad \sum_{i=1}^{3} x_{i2} = 11, \quad \sum_{i=1}^{3} x_{i3} = 8, \quad \sum_{j=1}^{3} x_{1j} = 12, \sum_{j=1}^{3} x_{2j} = 7, \quad \sum_{j=1}^{3} x_{3j} = 8$$

where $0 \leq x_{ij}$ $i = 1,2,3$; $j = 1,2,3$ and $\eta \geq 0$.

Also, using a hyperbolic membership function as defined in equation (3), the crisp model can be presented as follows:

$maximize: \eta$

$2.82x_{11} + 4.59x_{12} + 2.47x_{13} + 3.18x_{21} + 5.94x_{22} + 2.18x_{23} + 3.53x_{31} + 4.94x_{32} + 3.18x_{33} + x_{10} \leq 98.65$;
$1.56x_{11} + 2.18x_{12} + 1.70x_{13} + 1.78x_{21} + 2.46x_{22} + 1.2x_{23} + 1.80x_{31} + 2.48x_{32} + 1.78x_{33} + x_{10} \leq 51.31$;
Subject to.

$$\sum_{i=1}^{3} x_{i1} = 7, \quad \sum_{i=1}^{3} x_{i2} = 11, \quad \sum_{i=1}^{3} x_{i3} = 8, \quad \sum_{j=1}^{3} x_{1j} = 12, \sum_{j=1}^{3} x_{2j} = 7, \quad \sum_{j=1}^{3} x_{3j} = 8$$

where $0 \leq x_{ij}$, $i = 1,2,3$; $j = 1,2,3$ and $x_{10} \geq 0$.

Step 5. The optimal solution is obtained as follows:

i. The optimal solution for linear membership function is, $x^* = x_{11} = 7.0, x_{13} = 5.0,\ x_{22} = 3.0,\ x_{23} = 4.0,\ x_{32} = 8.0$ and hence $Z_1^* = 272,\ Z_2^* = 10573$ and $\eta = 0$.

ii. The optimal solution for exponential membership function using LINGO software is,

$$x^* = \begin{cases} x_{11} = 7, x_{12} = 0.8913946,\ x_{13} = 4.108605 \\ x_{22} = 2.450129, x_{23} = 4.549871, x_{32} = 7.658476, x_{33} = 0.3415239 \end{cases}$$

Hence $Z_1^* = 270.6919049,\ Z_2^* = 10469.31032$ and $\eta = 0$.

iii. The optimal solution for hyperbolic membership function using LINGO software is,

$$x^* = \begin{cases} x_{11} = 7.0, x_{12} = 0.1666667,\ x_{13} = 4.833333 \\ x_{22} = 2.833333, x_{23} = 4.166667, x_{32} = 8.0 \end{cases}$$

Hence $Z_1^* = 271.4999951,\ Z_2^* = 10546.33315$ and $x_{10} = 0$.

4. COMPARISON BETWEEN PROPOSED AND IDEAL SOLUTION OF BOTP:

For proving the effectiveness of our proposed algorithm, we can solve the BOTP as single-objective problem by taking one of the objectives at a time which gives an ideal solution.

	Proposed solution		**Ideal solution**	
$\boldsymbol{x}^{(1)}$	288	11169	263	10093
$\boldsymbol{x}^{(2)}$	271	9935	271	9935

5. CONCLUSION:

From the above analysis, it is analyzed that the proposed method for solving BOTP gives an efficient solution with less calculation and in simple way. The proposed technique has been illustrated with fish export case study. Here MOTP is used to solve two special types of linear and non-linear membership functions are used to solve the MOTP. From the obtained result, it is observed that the crisp model becomes non-linear when we use the exponential type membership function for different values of s. Here, if we compare the solution obtained by linear function, the optimal compromise solution does not change significantly. Moreover, the crisp model becomes linear, if we use the hyperbolic membership function. The proposed solution does not change if we compare the solution with linear function. The method has been solved with LINGO 17.0 software. This method is more suitable to apply for each decision maker.

Conflict of Interests

The authors have declared that no Conflict of Interest exists.

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