Special Type Labeling for the Extended Duplicate Graph of Quadrilateral Snake Graph

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Abstract - ***In this paper, we are discuss about that the Duplicate graph of Quadrilateral Snake $DG(QS_m)$, On Signed Product labeling, Sum Difference Cordial labeling, Sum Divisor Cordial labeling and Prime Cordial labeling are admits.***

Key words - **Graph Labeling, Duplicate graph, Quadrilateral snake, On Signed Product, Prime Cordial, Sum divisor, Sum Difference.**

INTRODUCTION

The concept of graph labeling was introduced by Rosa in 1967. A graph labeling is an assignment of integers to the vertices or edges or both subject to certain condition(s). E. Sampath kumar introduced the concept of duplicate graph in 1974. P.Vijaya kumar P.P Ulaganathan and K. Thirusangu, have proved that the duplicate graph of ladder graph is cordial, total cordial and prime cordial. Jayapal Baskar Babujee, Shobana Loganathan they also proved a "On Signed product Cordial Labeling" , Applied Mathematics, 2011, 1525-1530 Scientific Research. A. Lourdusamy. and F. Patrick have proved by graceful labeling of "Sum divisor cordial labeling for star and ladder related graphs", Proyecciones Journal of Mathematics Vol.No. 35,N^0 4, pp.437-455.

PRELIMINARIES

In this Article, we give the basic notions relevant to this Article.

Definition 1.1

Let G (V, E) be a simple graph. A duplicate graph of G is DG = (V_1, E_1), where the vertex set $V_1 = V \cup V'$ and $V \cap V' = \phi$ and f: $V \to V'$ is bijective (for $v \in V$, we write f(v) = v′ for convenience) and the edge set E_1 of DG is defined as: The edge uv is in E if and only if both uv′ and u′v are edges in E_1.

Definition: 1.2

A function $f : V \to \{0,1\}$ such that each edge uv receives the label $f(u) * f(v)$ is said to be Product cordial labeling. If the number of vertices labeled 0 and the number vertices labeled 1 differ by at most one, the number of edges labeled 0 and the number of edges labeled 1 differ by at most one.

Definition: 1.3

A vertex labeling of graph G $f: V \to \{-1,1\}$ with induced edge labeling $f(u) * f(v)$ defined by $f^*(uv) * f(u)f(v)$ is called on signed product cordial labeling if $|v_i(-1) - v_j(1)| \leq 1$ and $|e_i(-1) - e(1)| \leq 1$,where $v_i(-1)$ is the number of vertices labeled with -1, $v_j(1)$ is the number of vertices labeled with 1, $e_i(-1)$ is the number of edges labeled with -1 and $e_j(1)$ is the number of vertices labeled with 1 differ at most by one.

Definition: 1.4

A prime cordial labeling of a graph G with vertex set V is a bijection $f: V \to \{1, 2, 3, 4 \dots \dots |V|\}$ such that each edge uv is assigned the label as $f(uv) = \begin{Bmatrix} 1 \text{if g.c.d } \{f(u), f(v)\} = 1 \\ 0 \text{if g.c.d } \{f(u), f(v)\} > 1 \end{Bmatrix}$, and the number of edges labeled with '0'and the number of edges labeled with '1' differ at most by one.

Definition: 1.5

A sum divisor cordial labeling of a graph G with vertex set V is a bijection $f: V \to \{1, 2, 3, 4 \dots \dots |V|\}$ such that each edge uv is assigned the label 1 if $2 \mid [(f(u + f(v))]$ and the label 0 otherwise, and the number of edges labeled with '1'and the number of edges labeled with '0' differ at most by one.

Definition: 1.6

A Sum and Difference Prime Cordial labeling of a graph G with vertex set V is a bijection $f: V \to \{1, 2, 3, 4 \dots \dots |V|\}$ such that each edge uv is assigned the label as

$$f(uv) = \begin{Bmatrix} 1 \text{if g.c.d } \{f(u) + f(v), f(u) - f(v)\} = 1 \\ 0 \text{if g.c.d } \{f(u) + f(v), f(u) - f(v)\} > 1 \end{Bmatrix},$$

and the number of edges labeled with '0'and the number of edges labeled with '1' differ at most by one.

Definition: 1.7

The Quadrilateral snake QS_n is obtained from the path $\{u_1, u_2 \dots \dots u_n\}$ by joining u_i and u_{i+1} to two vertices v_i and w_i respectively and then joining v_i and w_i $1 \leq i \leq n-1$, where m is the number edges of a path by a circle C_4.its connected and undirected graph with 3m +1 vertices and 4m edges.

Definition: 1.8

Let $DG = (V_1, E_1)$ be a duplicate graph of the path graph $G(V, E)$. we add an edge between any one vertex from V to any other vertex in 'V'. For convenience, let us take $v_2 \in V$ and $v_2' \in V'$ and thus edge (v_2, v'_2) is formed. This graph is called the Extended Duplicate Graph of the path P_n and it's denoted by$EDG(p_m)$.

II.MAIN RESULTS

Algorithm 2.1: Construction of duplicate graph of Quadrilateral snake $DG(QS_m)$, $m \geq 2$

$V \to \{v_1, v_2, v_3 \dots \dots, v_m. v_1', v_2', \dots \dots v_m'\}$ $E \to \{e, e_2, e_3 \dots \dots, e_{4m}. e_1', e_2', \dots \dots e_{4m}'\}$

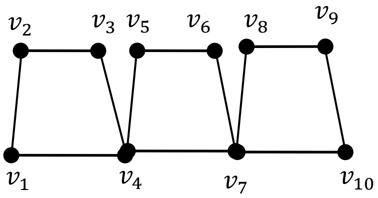
For $1 \leq i \leq m$

$v_{3i-2}v'_{3i+1} \leftarrow e_{4i-2}$ $v'_{3i-2}\, v_{3i+1} \leftarrow e'_{4i-1}$ $v_{3i-2}\, v'_{3i-1} \leftarrow e_{4i-3}$ $v'_{3i-2}\, v_{3i-1} \leftarrow e'_{4i-3}$

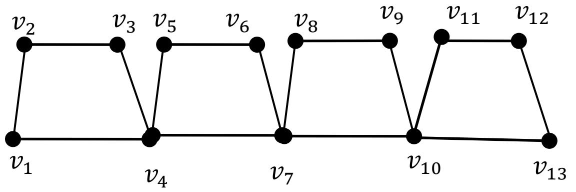
$v_{3i}\, v'_{3i+1} \leftarrow e_{4i}$ $v'_{3i}\, v_{3i+1} \leftarrow e'_{4i}$ $v_{3i-1}\, v'_{3i} \leftarrow e_{4i-1}$ $v'_{3i-1}v_{3i} \leftarrow e'_{4i-2}$

ILLUSTRATION:

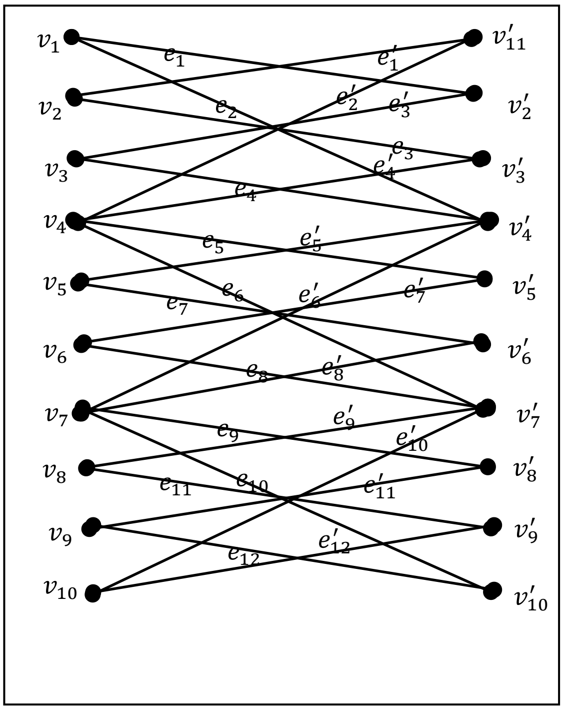
The following figures show the Quadrilateral snake graph and its Duplicate graph.



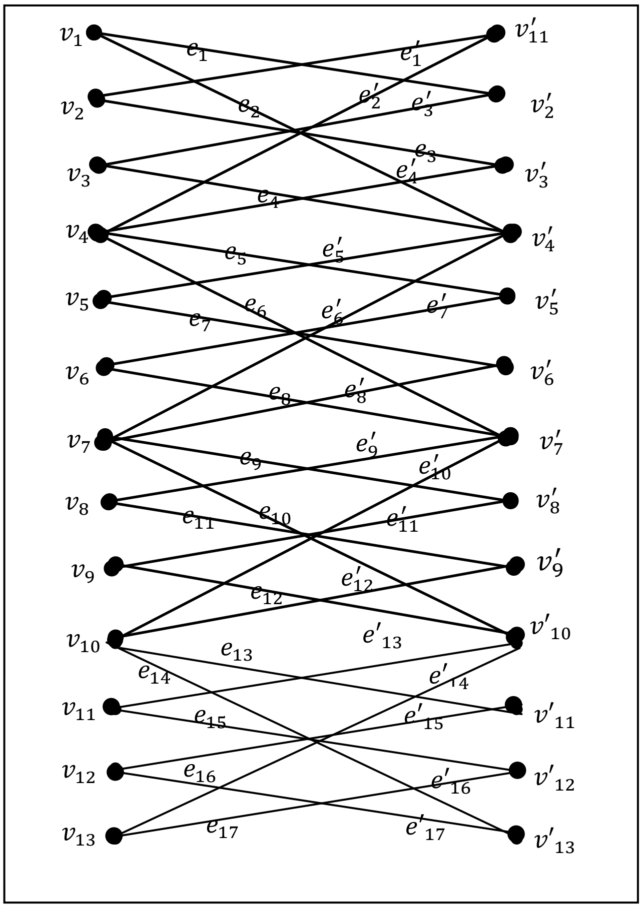
DG(QS_3)



DG(QS_4)



Remark: Duplicate graph of Quadrilateral Snake $DG(\,QS_m\,)$ $m \geq 2$, its contains 6m+2 vertices and 8m edges.



On Signed Product labeling $DG(\,QS_m\,)$

Algorithm 2.2: Assignment of labels to vertices

$V \rightarrow \{v_1, v_3 \ldots\ldots, v_{3m+1}. v_1', v_2', \ldots \ldots . v'_{3m+1}\}\ E \rightarrow \{e, e_2, e_3 \ldots\ldots, e_{4n}. e_1', e_2', \ldots \ldots . e'_{4m}\}$

For $1 \leq i \leq m$

Fix $v_1 \leftarrow 1$ $v'_1 \leftarrow$ -1

$v_{3i-1} \leftarrow -1\ v_{3i} \leftarrow 1.\ v_{3i+1} \leftarrow -1\ v'_{3i-1} \leftarrow 1, v'_{3i} \leftarrow 1\ v'_{3i+1} \leftarrow -1.$

Theorem 1.1: The duplicate graph of Quadrilateral snake $EDG(\,QS_m\,)$, $m \geq 2$, is on Signed Product Cordial labeling.

Proof: Let

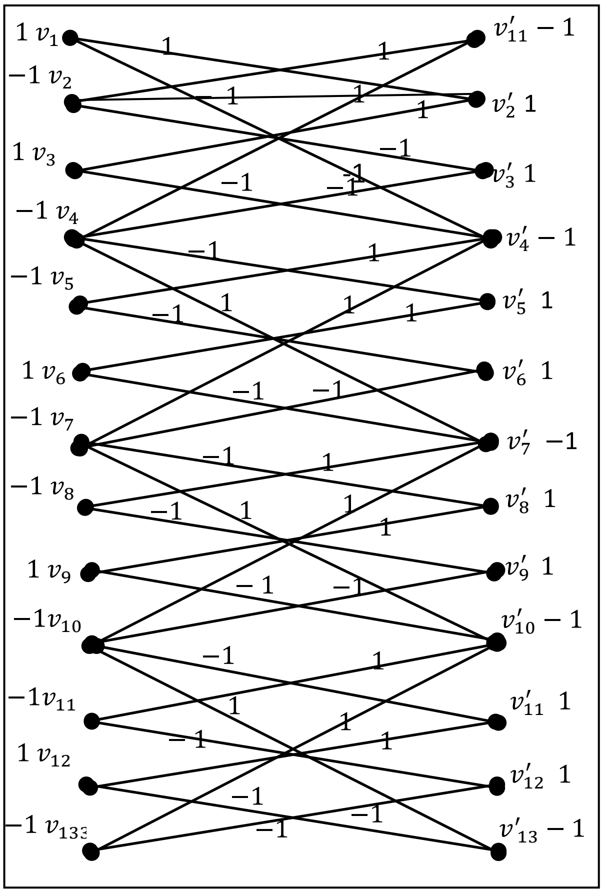
Let $V \rightarrow \{v_1, v_2, \ldots\ldots, v_{3m+1}. v_1', v_2', \ldots \ldots . v'_{3m+1}\}$,

$E \rightarrow \{e, e_2, e_3 \ldots\ldots, e_{4m}. e_1', e_2', \ldots \ldots . e'_{4m}\}$ be the set of vertices and edges of the duplicate graph of Quadrilateral graph $EDG(\,QS_m\,)$.using the algorithm 2.2, each of the 3m+1 vertices receive label -1 and 1 respectively. Using the induced function f^* defined by $f^*(uv) = f(u) * f(v)$, the 2m edges namely$e_3, e_4, e_7, e_8, \ldots \ldots . e_{4m}$.receive label -1 , the m-1 edges namely $e_5, e_9, e_{13}, e_{17}, e_{21}, \ldots \ldots \ldots \ldots, e_{4m-3}$. receive label -1, the 1 edges namely e_2 receive label -1. m edges namely $e'_4, e'_8, e'_{12}, e_{15}, \ldots \ldots \ldots \ldots . \ldots \ldots \ldots \ldots . e'_{4m}$. receive label -1.

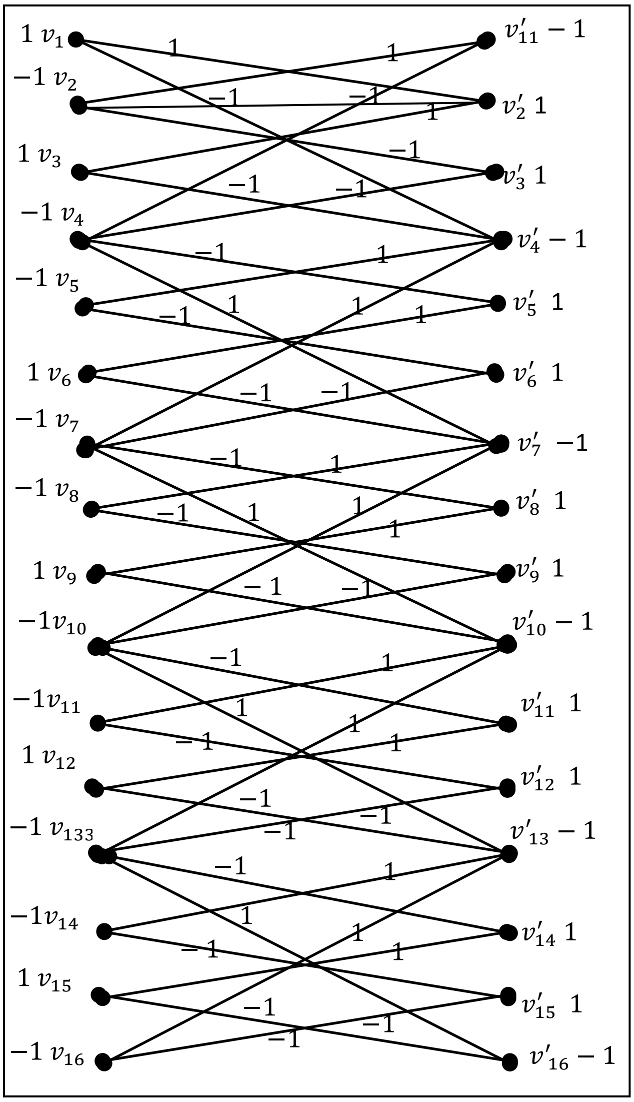
Thus 4m edges receive label -1. and the m-1 edges namely $e_6, e_{10}, e_{14}, e_{18}, e_{22}, \ldots \ldots \ldots \ldots . e_{4m-2}$. receive label 1, the 3m edges namely $e'_1, e'_2, e'_3, e'_5, e'_6, e'_7, , \ldots \ldots \ldots \ldots . \ldots \ldots \ldots \ldots . e_{4m-1}$ receive label 1, the 1 edges namely e_1 receive label 1. Thus the 4m edges receive label 1.

ILLUSTRATION:

$DG(QS_4)$



$DG(QS_5)$



Hence the duplicate graph of the Quadrilateral snake $DG(QS_m)$, $\boldsymbol{m \geq 2}$, signed Product Cordial labeling are admits.

Sum Difference cordial labeling $\boldsymbol{DG(\,QS_m\,)}$

Algorithm 2.3: Assignment of labels to vertices

$V \rightarrow \{v_1, v_2, v_3 \ldots\ldots, v_{3m+1}. v_{1'}, v_{2'}, \ldots \ldots . v'_{3m+1}\}$

$E \rightarrow \{e, e_2, e_3 \ldots\ldots, e_{4n}. e_{1'}, e_{2'}, \ldots \ldots . e'_{4m}\}$

Case (ii): when m is Even

For $1 \leq i \leq \frac{m}{2}$

$v_{6i-5} \leftarrow 12i - 11$: $v_{6i-4} \leftarrow 12i - 9$: $v_{6i-3} \leftarrow 12i - 6$: $v_{6i-2} \leftarrow 12i - 4$: $v_{6i-1} \leftarrow 12i - 2$ $v_{6i} \leftarrow 12i - 1$: $v'_{6i-5} \leftarrow 12i - 10$: $v'_{6i-4} \leftarrow 12i - 8$: $v'_{6i-3} \leftarrow 12i - 7$: $v'_{6i-2} \leftarrow 12i - 5$: $v'_{6i-1} \leftarrow 12i - 3$: $v'_{6i} \leftarrow 12i$

For $i = m$

$v_{3i+1} \leftarrow 6i + 1$ $v'_{3i+1} \leftarrow 6i + 2$

Cae (ii): when m is Odd

For $1 \leq i \leq \frac{m+1}{2}$

$v_{6i-5} \leftarrow 12i - 11$: $v_{6i-4} \leftarrow 12i - 9$: $v_{6i-3} \leftarrow 12i - 6$: $v_{6i-2} \leftarrow 12i - 4$:

$v'_{6i-5} \leftarrow 12i-10$: $v'_{6i-4} \leftarrow 12i-8$: $v'_{6i-3} \leftarrow 12i-7$: $v'_{6i-2} \leftarrow 12i-5$

For $1 \le i < \frac{m+1}{2}$

$v_{6i-1} \leftarrow 12i-2$: $v_{6i} \leftarrow 12i-1$: $v'_{6i-1} \leftarrow 12i-3$: $v'_{6i} \leftarrow 12i$

Theorem 1.4: The duplicate graph of Quadrilateral snake $\boldsymbol{DG(QS_m)}$, $m \ge 2$, is Sum Difference Cordial labeling.

Proof:

Let $V \rightarrow \{v_1, v_2, \ldots\ldots, v_{3m+1}. v_1', v_2', \ldots\ldots. v'_{3m+1}\}$, $E \rightarrow \{e, e_2, e_3 \ldots\ldots, e_{4m}. e_1', e_2', \ldots\ldots. e'_{4m}\}$ be the set of vertices and edges of the duplicate graph of Quadrilateral graph $EDG(QS_m)$.using the algorithm 2.3, each of the 3m+1 vertices receive label 0 and 1 respectively.

Case (i): when m is even

For $1 \le i \le \frac{m}{2}$, the $\left(\frac{m}{2}\right)$ vertices v_{6i-5} receive label $12i-11$, the $\left(\frac{m}{2}\right)$ vertices v_{6i-4} receive label $12i-9$, the $\left(\frac{m}{2}\right)$ vertices v_{6i-3} receive label $12i-6$, the $\left(\frac{m}{2}\right)$ vertices v_{6i-2} receive label $12i-4$, the $\left(\frac{m}{2}\right)$ vertices v_{6i-1} receive label $12i-2$, the $\left(\frac{m}{2}\right)$ vertices v_{6i} receive label $12i-1$, For $i = m$, vertex v_{3i+1} receive $6i+1$ vertex v'_{3i+1} receive $6i+2$. the $\left(\frac{m}{2}\right)$ vertices v'_{6i-5} receive label $12i-10$, the $\left(\frac{m}{2}\right)$ vertices v'_{6i-4} receive label $12i-8$, the $\left(\frac{m}{2}\right)$ vertices v'_{6i-3} receive label $12i-7$, the $\left(\frac{m}{2}\right)$ vertices v'_{6i-2} receive label $12i-5$, the $\left(\frac{m}{2}\right)$ vertices v'_{6i-1} receive label $12i-3$, the $\left(\frac{m}{2}\right)$ vertices v'_{6i} receive label $12i$, For $i = m$, vertex v_{3i+1} receive $6i+1$ vertex v'_{3i+1} receive $6i+2$. Thus the entire 6m+2 vertices are labeled.

The induced function $f^* : E \rightarrow \{0,1\}$ is defined such that $f: V \rightarrow \{1, 2, 3, 4 \ldots\ldots |V|\}$ such that each edge

$$f(uv) = \begin{cases} 1 \text{if g.c.d } \{f(u)+f(v), f(u)-f(v)\} = 1 \\ 0 \text{if g.c.d } \{f(u)+f(v), f(u)-f(v)\} > 1 \end{cases}$$

The m edges namely $e_1, e_5, e_9, e_{13}, e_{17}, e_{21}, \ldots\ldots\ldots\ldots\ldots e_{4m-3}$ receive label 1. The m edges namely $e_4, e_8, e_{12}, e_{16}, e_{20}, e_{24}, \ldots\ldots\ldots\ldots\ldots e_{4m}$. receive label 1. The m edges namely $e'_1, e'_5, e'_9, e'_{13}, e'_{17}, e'_{21}, \ldots\ldots\ldots\ldots e'_{4m-3}$. receive label 1. The m edges namely $e'_4, e'_8, e'_{12}, e'_{16}, e'_{20}, e'_{24}, \ldots\ldots\ldots\ldots e'_{4m}$. receive label 1.

Thus 4m edges receive label 1. and the 2m edges namely $e_2, e_3, e_6, e_7, e_{10}, e_{11}, \ldots\ldots\ldots\ldots e_{4m-2}, e_{4m-1}$. receive label 0, 2m edges namely $e'_2, e'_3, e'_6, e'_7, e'_{10}, e'_{11}, \ldots\ldots\ldots\ldots e'_{4m-2}, e'_{4m-1}$. receive label 0Thus the 4m edges receive label 0.

Case (ii) when m is Odd

For $1 \le i \le \frac{m+1}{2}$, the $\left(\frac{m+1}{2}\right)$ vertices v_{6i-5} receive label $12i-11$, the $\left(\frac{m+1}{2}\right)$ vertices v_{6i-4} receive label $12i-9$, the $\left(\frac{m+1}{2}\right)$ vertices v_{6i-3} receive label $12i-6$, the $\left(\frac{m+1}{2}\right)$ vertices v_{6i-2} receive label $12i-4$, the $\left(\frac{m+1}{2}\right)$ vertices v'_{6i-5} receive label $12i-10$, the $\left(\frac{m+1}{2}\right)$ vertices v'_{6i-4} receive label $12i-8$, the $\left(\frac{m+1}{2}\right)$ vertices v'_{6i-3} receive label $12i-7$, the $\left(\frac{m+1}{2}\right)$ vertices v'_{6i-2} receive label $12i-5$, For $1 \le i < \frac{m+1}{2}$, the $\left(\frac{m+1}{2}\right)$ vertices v_{6i-1} receive label $12i-2$, the $\left(\frac{m+1}{2}\right)$ vertices v_{6i} receive label $12i-1$, the $\left(\frac{m+1}{2}\right)$ vertices v'_{6i-1} receive label $12i-3$, the $\left(\frac{m+1}{2}\right)$ vertices v'_{6i} receive label 6i. Thus the entire 6m+2 vertices are labeled.

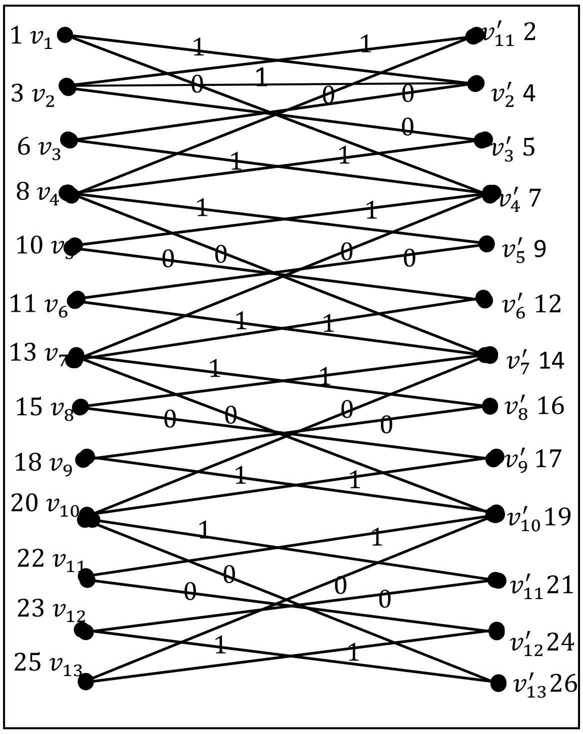
The induced function $f^* : E \rightarrow \{0,1\}$ is defined such that $f: V \rightarrow \{1, 2, 3, 4 \ldots\ldots |V|\}$ such that each edge

$$f(uv) = \begin{cases} 1 \text{if g.c.d } \{f(u)+f(v), f(u)-f(v)\} = 1 \\ 0 \text{if g.c.d } \{f(u)+f(v), f(u)-f(v)\} > 1 \end{cases},$$

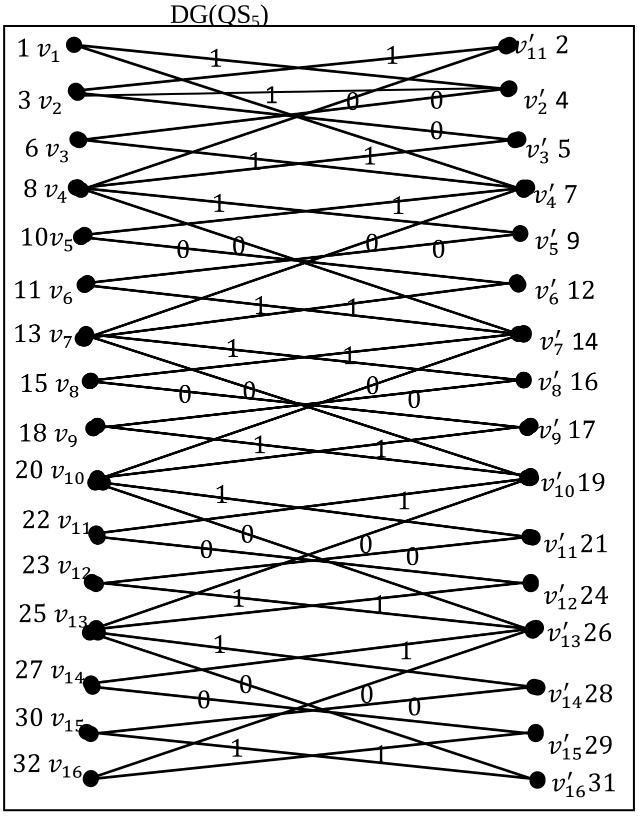
the m edges namely $e_1, e_5, e_9, e_{13}, e_{17}, e_{21}, \ldots\ldots\ldots\ldots\ldots e_{4m-3}$. receive label 1, the m edges namely $e_4, e_8, e_{12}, e_{16}, e_{20}, e_{24}, \ldots\ldots\ldots\ldots\ldots e_{4m}$. receive label 1, the m edges namely $e'_1, e'_5, e'_9, e'_{13}, e'_{17}, e'_{21}, \ldots\ldots\ldots\ldots e'_{4m-3}$. receive label 1, the m edges namely $e'_4, e'_8, e'_{12}, e'_{16}, e'_{20}, e'_{24}, \ldots\ldots\ldots\ldots e'_{4m}$. receive label 1.Thus 4m edges receive label 1. and the 2m edges namely $e_2, e_3, e_6, e_7, e_{10}, e_{11}, \ldots\ldots\ldots\ldots e_{4m-2}, e_{4m-1}$. receive label 0, 2m edges namely $e'_2, e'_3, e'_6, e'_7, e'_{10}, e'_{11}, \ldots\ldots\ldots\ldots e'_{4m-2}, e'_{4m-1}$. receive label 0. Thus the 4m edges receive label 0.

ILLUSTRATION:

DG(QS_4)



DG(QS_5)



Hence, the duplicate graph of the quadrilateral snake $DG(QS_m)$, $m \geq 2$, is Sum Difference Cordial labeling are admits.

Sum Divisor Cordial labeling $DG(QS_m)$

Algorithm 2.4: Assignment of labels to vertices

$V \rightarrow \{v_1, v_2, v_3 \ldots\ldots, v_{3m+1}. v_{1'}, v_{2'}, \ldots\ldots. v'_{3m+1}\}, E \rightarrow \{e, e_2, e_3 \ldots\ldots, e_{4n}. e_{1'}, e_{2'}, \ldots\ldots. e'_{4m}\}$

Case (i): when m is Even

For $1 \leq i \leq m+1$

$v_{3i-2} \leftarrow 6i-4$: $v_{3i-1} \leftarrow 6i-3$: $v_{3i} \leftarrow 6i-1$ $v'_{3i-2} \leftarrow 6i-5$: $v'_{3i-1} \leftarrow 6i-2$: : $v'_{3i} \leftarrow 6i$

For $i = m$

$v_{3i+1} \leftarrow 6i+2$ $v'_{3i+1} \leftarrow 6i+1$

Case (ii): when m is Odd

For $1 \leq i \leq m$

$v_{3i-2} \leftarrow 6i-4$: $v_{3i-1} \leftarrow 6i-3$: $v_{3i} \leftarrow 6i-1$: $v'_{3i-2} \leftarrow 6i-5$: $v'_{3i-1} \leftarrow 6i-2$ $v'_{3i} \leftarrow 6i$

For $i = m$

$v_{3i+1} \leftarrow 6i+2$ $v'_{3i+1} \leftarrow 6i+1$

Theorem 1.3: The duplicate graph of Quadrilateral snake $\boldsymbol{DG(QS_m)}$, $m \geq 2$, is Sum Divisor Cordial labeling.

Proof:

Let $V \rightarrow \{v_1, v_2, \ldots\ldots, v_{3m+1}. v_{1'}, v_{2'}, \ldots\ldots. v'_{3m+1}\}$, $E \rightarrow \{e, e_2, e_3 \ldots\ldots, e_{4m}. e_{1'}, e_{2'}, \ldots\ldots. e'_{4m}\}$ be the set of vertices and edges of the duplicate graph of Quadrilateral graph $EDG(QS_m)$.using the algorithm 2.4, each of the 3m+1 vertices receive label 0 and 1 respectively.

Case (i): when m is even

For $1 \leq i < m+1$, the $(m+1)$ vertices v_{3i-2} receive label $6i-4$, the $(m+1)$ vertices v_{3i-1} receive label $6i-3$, the $(m+1)$ vertices v_{3i} receive label $6i-1$, the $(m+1)$ vertices v'_{3i-2} receive label $6i-5$, the

$(m+1)$ vertices v'_{3i-1} receive label $6i-2$, the $(m+1)$ vertices v'_{3i} receive label $6i$. For $i=m$, vertex v_{3i+1} receive $6i+1$ vertex v'_{3i+1} receive $6i+2$. Thus the entire 6m+2 vertices are labeled.

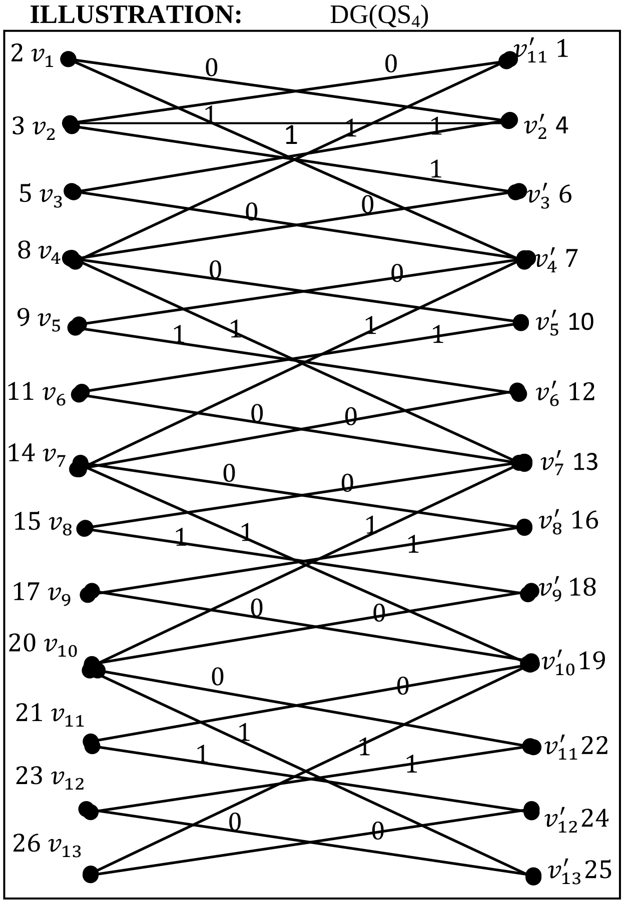
The induced function $f^*: E \to \{0,1\}$ is defined such that $f: V \to \{1, 2, 3, 4 \dots\dots |V|\}$ such that each edge uv is assigned the label 1 if $2 \mid [(f(u+f(v))]$ and the label 0 otherwise, the 2m edges namely e_2, e_3, e_6, e_7, e_{10}, e_{11}, $\dots\dots\dots\dots e_{4m-1}$. receive label 1, the 2m edges namely $e'_2, e'_3, e'_6, e'_7, e'_{10}, e'_{11}, \dots\dots\dots\dots e'_{4m-1}$. receive label 1.Thus 4m edges receive label 1. and m edges namely e_4, e_5, e_{12}, e_{13}, $\dots\dots\dots\dots, e_{4m-4}, e_{4m-3}$ .receive label 0, the $\frac{m}{2}$ edges namely $e_1, e_9, e_{17}, \dots, e_{4m-7}$. receive label 0, the $\frac{m}{2}$ edges namely $e_8, e_{16}, e_{24}, \dots, e_{4m}$. receive label 0, and the m edges namely e'_4, e'_5, e'_{12}, e'_{13}, $\dots\dots\dots\dots, e'_{4m-4,} e'_{4m-3}$ .receive label 0, the $\frac{m}{2}$ edges namely $e'_1, e'_9, e'_{17}, \dots, e'_{4m-7}$. receive label 0, the $\frac{m}{2}$ edges namely $e'_8, e'_{16}, e'_{24}, \dots\dots\dots, e'_{4m}$. receive label 0, Thus the 4m edges receive label 0.

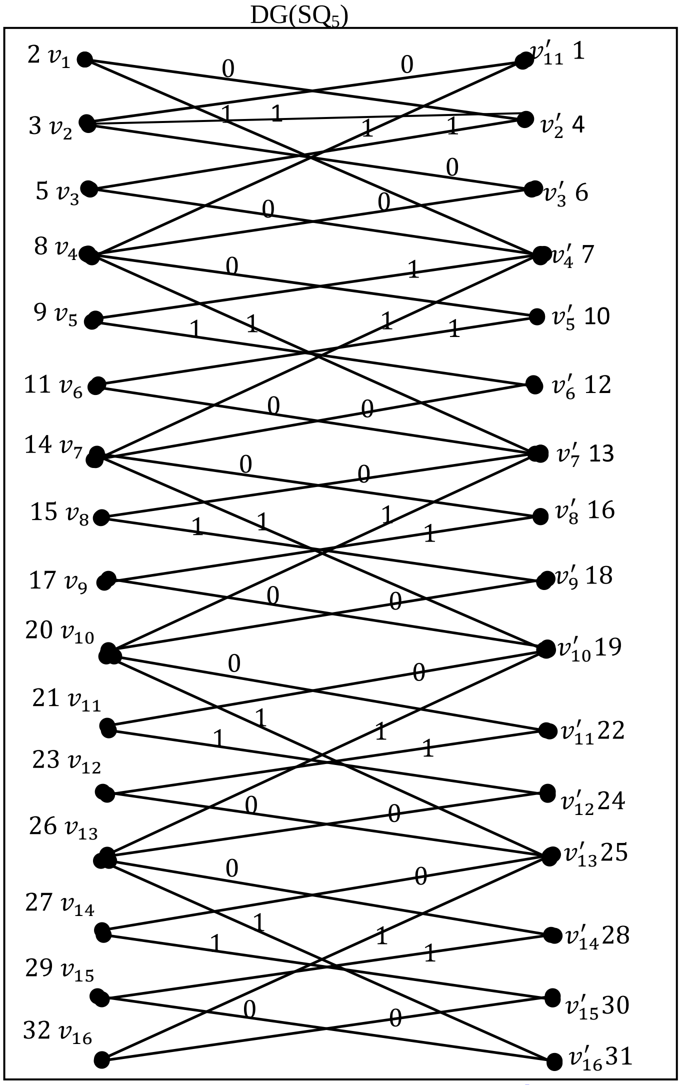
Case (ii): when m is Odd

For $1 \le i \le m$, the (m) vertices v_{3i-2} receive label $6i-4$, the (m) vertices v_{3i-1} receive label $6i-3$, the (m) vertices v_{3i} receive label $6i-1$, the (m) vertices v'_{3i-2} receive label $6i-5$, the (m) vertices v'_{3i-1} receive label $6i-2$, the (m) vertices v'_{3i} receive label $6i$. For $i=m$, vertex v_{3i+1} receive $6i+2$ vertex v'_{3i+1} receive $6i+1$. Thus the entire 6m+2 vertices are labeled.

The induced function $f^*: E \to \{0,1\}$ is defined such that $f: V \to \{1, 2, 3, 4 \dots\dots |V|\}$ such that each edge uv is assigned the label 1 if $2 \mid [(f(u+f(v))]$ and the label 0 otherwise, the 2m edges namely e_2, e_3, e_6, e_7, e_{10}, e_{11}, $\dots\dots\dots\dots e_{4m-1}$. receive label 1, the 2m edges namely $e'_2, e'_3, e'_6, e'_7, e'_{10}, e'_{11}, \dots\dots\dots\dots e'_{4m-1}$. receive label 1.Thus 4m edges receive label 1. and m edges namely $e_1, e_5, e_9, e_{13}, \dots\dots\dots\dots e_{4m-3}$.receive label 0, the m edges namely $e_4, e_8, e_{12}, e_{20}, e_{24}, \dots, e_{4m}$. receive label 0, and the m edges namely $e'_1, e'_5, e'_9, e'_{13}, \dots\dots\dots\dots e'_{4m-3}$ receive label 0, m edges namely $e'_4, e'_8, e'_{12}, e'_{20}, e'_{24}, \dots, \dots e'_{4m}$.receive label 0. Thus the 4m edges receive label 0.

ILLUSTRATION: DG(QS$_4$) DG(SQ$_5$)





Hence, the duplicate graph of the quadrilateral snake $DG(QS_m)$, $m \ge 2$, is Sum Divisor Cordial labeling are admits.

Prime cordial labeling $DG(QS_m)$

Algorithm 2.5: Assignment of labels to vertices

$V \rightarrow \{v_1, v_2, v_3 \ldots\ldots, v_{3m+1}. v_1', v_2', \ldots\ldots. v_{3m+1}'\}$
$E \rightarrow \{e, e_2, e_3 \ldots\ldots, e_{4n}. e_1', e_2', \ldots\ldots. e_{4m}'\}$

Case (i): when m is odd

For $1 \leq i \leq \frac{3m+1}{2}$
$v_{2i} \leftarrow 4i$: $v_{2i-1} \leftarrow 4i-3$ $v'_{2i-1} \leftarrow 4i-2$: $v'_{2i} \leftarrow 4i-1$

Case (ii): when m is even

For $1 \leq i \leq \frac{3m}{2}$
$v_{2i} \leftarrow 4i$: $v_{2i-1} \leftarrow 4i-3$ $v'_{2i-1} \leftarrow 4i-2$: $v'_{2i} \leftarrow 4i-1$
For $i = m$
$v_{3i+1} \leftarrow 6i+1$ $v'_{3i+1} \leftarrow 6i+2$

Theorem 1.2: The duplicate graph of Quadrilateral snake $DG(QS_m)$, $m \geq 2$, is prime cordial.

Proof:
Let $V \rightarrow \{v_1, v_2, \ldots\ldots, v_{3m+1}. v_1', v_2', \ldots\ldots. v_{3m+1}'\}$, $E \rightarrow \{e, e_2, e_3 \ldots\ldots, e_{4m}. e_1', e_2', \ldots\ldots. e_{4m}'\}$ be the set of vertices and edges of the duplicate graph of Quadrilateral graph $EDG(QS_m)$.using the algorithm 2.5, each of the 3m+1 vertices receive label 0 and 1 respectively.

Case (i): when m is odd

For $1 \leq i \leq \frac{3m+1}{2}$, the $\left(\frac{3m+1}{2}\right)$, vertices v_{2i} receive label $4i$, the $\left(\frac{3m+1}{2}\right)$ vertices v_{2i-1} receive label $4i-3$, the $\left(\frac{3m+1}{2}\right)$ vertices v'_{2i-1} receive label $4i$, the $\left(\frac{3m+1}{2}\right)$ vertices v'_{2i} receive label $4i$-1 Thus the entire 6m+2 vertices are labeled.

The induced function $f^* : E \rightarrow \{0,1\}$ is defined such that $f(uv) = \begin{Bmatrix} 1 \text{if g.c.d } \{f(u), f(v)\} = 1 \\ 0 \text{if g.c.d } \{f(u), f(v)\} > 1 \end{Bmatrix}$ Using the induced function, the m+1 edges namely $e_1, e_2, e_9, e_{10}, e_{17}, e_{18}, \ldots\ldots\ldots\ldots. e_{4m-3}, e_{4m-2}$. receive label 1 , the $\frac{m+1}{2}$ edges namely e_4 , e_{12} , e_{20} , e_{28} , $\ldots\ldots\ldots\ldots, e_{4m}$ .receive label 1 the $\frac{m-1}{2}$ edges namely $e_7, e_{15}, e_{23}, \ldots, e_{4m-5}$. receive label 1, the $\frac{m+1}{2}$ edges namely $e'_3, e'_{11}, e'_{19}, \ldots\ldots\ldots\ldots, e'_{4m-1}$. receive label 1. the m−1 edges namely $e'_5, e'_6, e'_{13}, e'_{13}, e'_{14}, \ldots\ldots\ldots\ldots e'_{4m-7}, e'_{4m-6}$. receive label 1, the $\frac{m-1}{2}$ edges namely $e'_8, e'_{16}, e'_{24}, \ldots\ldots\ldots\ldots, e'_{4m-4}$ receive label 1, Thus the 4m edges receive label 1. and the m+1 edges e'_1 , e'_2 , e'_9 , e'_{10} , e'_{17} , $e'_{18} \ldots\ldots e'_{4m-3}, e'_{4m-2}$. receive label 0 , the $\frac{m+1}{2}$ edges namely $e'_4, e'_{12}, e'_{20}, e'_{28}, \ldots\ldots\ldots\ldots, e'_{4m}$.receive label 0, the $\frac{m-1}{2}$ edges namely $e'_7, e'_{15}, e'_{23}, \ldots\ldots, e'_{4m-5}$. receive label 0, the $\frac{m+1}{2}$ edges namely e_3, e_{11}, e_{19}, $\ldots\ldots\ldots, e_{4m-1}$. receive label 0, the $m-1$ edges namely e_5 , e_6 , e_{13} , e_{13} , e_{14} , $\ldots\ldots\ldots\ldots e_{4m-7}, e_{4m-6}$. receive label 0, the $\frac{m-1}{2}$ edges namely $e_8, e_{16}, e_{24}, \ldots\ldots\ldots\ldots, e_{4m-4}$ receive label 0, Thus the number of edges with label 1, Thus the 4m edges receive label 0.

Case (ii): when m is Even

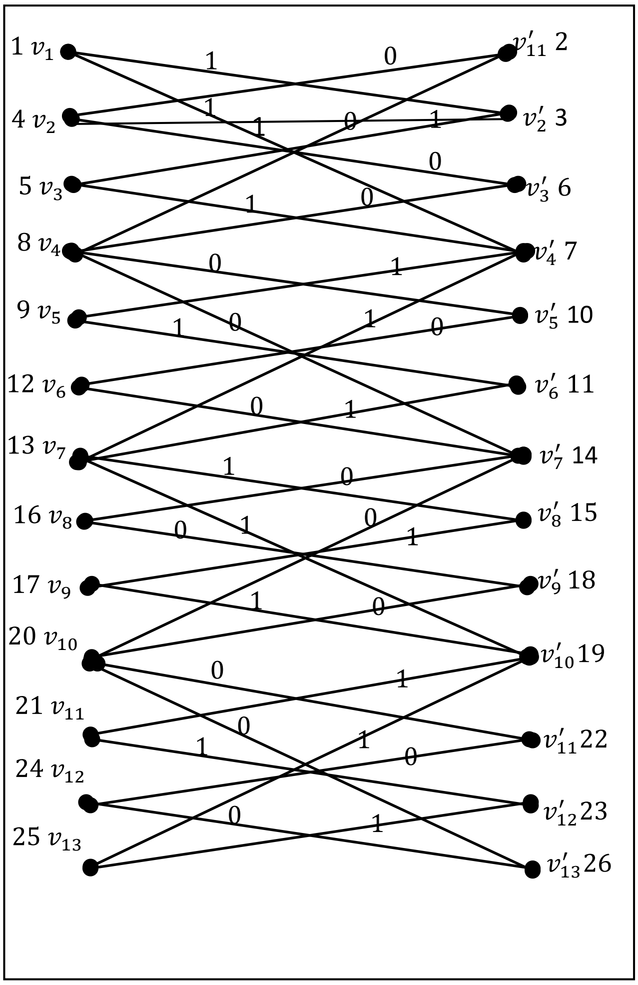
For $1 \leq i \leq \frac{3m}{2}$, the $\left(\frac{3m}{2}\right)$ vertices v_{2i} receive label $4i$, the $\left(\frac{3m+1}{2}\right)$ vertices v_{2i-1} receive label $4i-3$, the $\left(\frac{3m+1}{2}\right)$ vertices v'_{2i-1} receive label $4i$, the $\left(\frac{3m+1}{2}\right)$ vertices v'_{2i} receive label $4i$-1 For $i = m$, vertex v_{3i+1} receive $6i+1$ vertex v'_{3i+1} receive $6i+2$. Thus the entire 6m+2 vertices are labeled.

Using the induced function f^* defined by $f^*(uv) = f(u) * f(v)$, the m edges namely e_1 , e_2 , e_9 , e_{10} , e_{17} , e_{18} , …………..e_{3m-1}, e_{3m} . receive label 1 , the $\frac{m}{2}$ edges namely $e_4, e_{12}, e_{20}, e_{28}, \ldots\ldots\ldots, e_{4m-4}$.receive label 1, the $\frac{m}{2}$ edges namely $e_7, e_{15}, e_{23}, \ldots, e_{4m-1}$. receive label 1, the $\frac{m}{2}$ edges namely e'_3 , e'_{11} , e'_{19} , ……………, e'_{4m-5} . receive label 1, the m edges namely e'_5 , e'_6 , e'_{13} , e'_{13} , e'_{14} , …………..e'_{4m-3}, e'_{4m-2} . receive label 1, the $\frac{m}{2}$ edges namely $e'_8, e'_{16}, e'_{24}, \ldots\ldots\ldots, e_{4m}$ receive label 1, and the m edges $e'_1, e'_2, e'_9, e'_{10}, e'_{17}, e'_{18}$ ……..e'_{3m-1}, e'_{3m}. receive label 0 , the $\frac{m}{2}$ edges namely $e'_4, e'_{12}, e'_{20}, e'_{28}, \ldots\ldots\ldots, e'_{4m-4}$.receive label 0, the $\frac{m}{2}$ edges namely $e'_7, e'_{15}, e'_{23}, \ldots, e'_{4m-1}$. receive label 0, the $\frac{m}{2}$ edges namely $e_3, e_{11}, e_{19}, \ldots\ldots, e_{4m-5}$. receive label 0, the m edges namely e_5, e_6, e_{13}, e_{14}, ……………e_{4m-3}, e_{4m-2}. receive label 0, the $\frac{m}{2}$ edges namely $e_8, e_{16}, e_{24}, \ldots\ldots\ldots, e_{4m}$ receive label 0.

Thus the 4m edges receive label 0. Hence 6m+2 vertices are labeled and 8m edges received label 0 and 1 respectively.

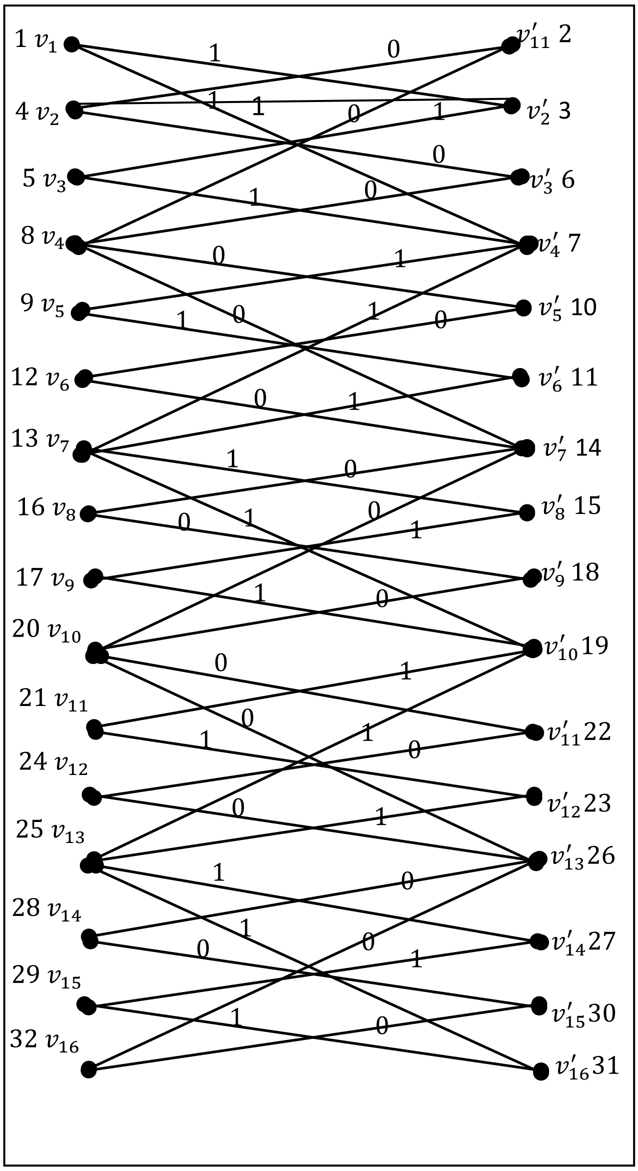
ILLUSTRATION:

DG(QS$_4$)



Hence, the duplicate graph of the quadrilateral snake $DG(QS_m)$, $m \geq 2$, is Prime Cordial are admits.

DG(SQ$_5$)



III. CONCLUSIONS

We proved that the duplicate graph of Quadrilateral snake $DG(QS_m)$, $m \geq 2$, On *Signed Product labeling, Sum Difference Cordial labeling, Sum Divisor Cordial labeling and Prime Cordial labeling.*

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