Algebraic Properties of Max Weighted Finite State Mealy Machines

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Abstract

The purpose of this paper is to introduce and study the concepts of weak homomorphism, homomorphism, strong homomorphism, identical states and admissible relations on Max weighted finite state mealy machine ($mwfmm$).

Keywords: Mealy machine, Max weighted finite state automaton, Max weighted finite state mealy machine, homomorphism, identical states, equivalence relation, admissible relations.

1 Introduction

Mathematical models in classical computation, automata have been an important area in theoretical computer science [5]. It started from the seminal papers of Kleene [7] and within a few years developed into a rich mathematical research topic. From the beginning finite automata constituted a core of computer science. Part of the reason is that they capture something very fundamental as it is witnessed by a numerous different characterizations of the family of rational languages, i.e., languages defined by finite automata [5]. Finite automata plays a crucial role in the theory of programming languages, compiler constructions, switching circuit designing, computer controllers,neuron net, text editor and lexical analyzer.[1].

Weighted automata were introduced by Schutzenberger(1961).Weighted finite automata are classical nondeterministic finite automata in which the transition carry weights. These weights may model e.g., the cost involved when executing a transition, the amount of resources or time needed for this, or probability or reliability of its successful execution. The behaviour of weighted finite automata can then be considered as the function (suitably defined) associating with each

word the weight of its execution. Clearly, weights can also be added to classical automata with infinite state sets like pushdown automata; this extension constitutes the general concept of weighted automata.
The main aim of this paper is to apply algebraic concepts on Max weighted finite state mealy machine. First we discuss some basic concepts related to Max weighted finite state automaton (mwfa) with appropriate examples, Mealy machine [2, 3] are defined. In section 3, the idea of Max weighted finite state mealy machine ($mwfmm$) has been grasped from [9], the definition is extended with suitable lemma. Weak homomorphism, homomorphism and strong homomorphism [4] on $mwfmm$ are defined in Section 4.
Section 5 covers identical states and in section 6 admissible relations on the set of states of a ($mwfmm$) is characterized. The Kernel of a homomorphism is defined and it is a admissible relation on Q is proved.

2 Preliminaries

This section introduces the notions of mwfa and a set recognized by a mwfa.
Definition 1. A Max weighted finite state automaton is a six tuple (mwfa) $M=(Q,\Sigma,W,\mu,i,f)$, where

(i) Q is a finite non-empty set of states.

(ii) Σ is a finite non-empty set of input symbols.

(iii) W is a weighting space. i.e., weighting space $W=([0,\infty),\cdot,\max)$, where $\cdot$ is usual multiplication.

(iv) μ is a weighting function such as $\mu : Q\times\Sigma\times Q \rightarrow [0,\infty)$ is called a state transition function. The value of $\mu(p,a,q)$ represents the weighted transition from state p to state q when the input symbol is a.

(v) i is an initial distribution function, where $i : Q\rightarrow[0,\infty)$.

(vi) f is a final distribution function, where $f : Q\rightarrow[0,\infty)$.

Definition 2. Let $M=(Q,\Sigma,W,\mu,i,f)$ be a mwfa, the extended weighting transition function for M is the weighted subset
$\mu^* : Q\times\Sigma^*\times Q \rightarrow [0,\infty)$ has defined as follows: $\forall p,q \in$Q, $a\in\Sigma$, $x\in\Sigma^*$

$$\mu^*(p,\lambda,q)=\begin{cases}1, \text{if } p=q\\ 0, \text{if } p\neq q\end{cases}$$
$$\mu^*(p,xa,q)=\max_{r\in Q}\{\mu^*(p,x,r)\cdot\mu(r,a,q)\}$$

Definition 3. Let $M=(Q,\Sigma,W,\mu,i,f)$ be a mwfa. Let $x\in\Sigma^*$. Then x is said to be recognized by M if $L(x)>0$, where

$$\begin{aligned}L(x)&=\max_{p,q\in Q}\{i(p)\cdot\mu^*(p,x,q)\cdot f(q)\}\\&=\max_{p,q\in Q}\Big\{i(p)\cdot\{\max_{r\in Q}\mu^*(p,y,r)\cdot\mu(r,a,q)\}\cdot f(q)\Big\},\ \ x=ya.\end{aligned}$$

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Theorem 1. Let $M = (Q, \Sigma, W, \mu, i, f)$ be a mwfa. Then
$\mu^*(p, xy, q) = \max_{r \in Q}\{\mu^*(p, x, r) \cdot \mu^*(r, y, q)\}\ \forall\ p, q \in Q$ and $\forall\ x, y \in \Sigma^*$.

Example 1. Let $M = (Q, \Sigma, W, \mu, i, f)$ be a mwfa, where $Q = \{q_1, q_2, q_3\}$, $\Sigma = \{0, 1\}$, $\mu : Q \times \Sigma \times Q \rightarrow [0, \infty)$ is defined as follows:

$$\begin{array}{lll} \mu(q_1, 0, q_1) = 2 & \mu(q_2, 0, q_3) = 6 & \mu(q_1, 1, q_1) = 4 \\ \mu(q_3, 0, q_3) = 2.5 & \mu(q_1, 1, q_2) = 3 & \mu(q_3, 1, q_3) = 3.5 \\ \mu(q_1, 0, q_2) = 5 & & \end{array}$$

we omit the weight values which are zero.
$i : Q \rightarrow [0, \infty)$ is defined by $i(q_1) = 2$, $i(q_2) = 3$.
$f : Q \rightarrow [0, \infty)$ is defined by $f(q_3) = 5$.
The language accepted by M is a weighted subset $L : \Sigma^* \rightarrow [0, \infty)$ such that

$$L(x) = \begin{cases} w_1, & w_1 \geq 180 \quad \text{if } x \in \{0,1\}^*10\{0,1\}^* \\ w_2, & w_2 \geq 300 \quad \text{if } x \in \{0,1\}^*00\{0,1\}^* \\ w_3, & w_3 \geq 90 \quad \text{if } x \in 0\{0,1\}^* \\ 0\ , & \text{otherwise} \end{cases}$$

Definition 4. A mealy machine is a six tuple $M = (Q, X, Y, q_0, T, G)$, where

(i) Q is a finite non-empty set of states.

(ii) X is a finite non-empty set of input symbols.

(iii) Y is a finite non-empty set of output symbols.

(iv) $q_0 \in Q$ is the initial state.

(v) $T : Q \times X \rightarrow Q$ is called a transition function.

(vi) $G : Q \times X \rightarrow Y$ is called a output function.

3 A Max weighted finite state mealy machine

In this section the idea of Max weighted finite state mealy machine has been grasped from [8], the definition is extended with suitable lemma.

Definition 5. A Max weighted finite state mealy machine ($mwfmm$) is a five tuple $\mathscr{M} = (Q, X, Y, W, \alpha)$, where

(i) Q is a finite non-empty set of states.

(ii) X is a finite non-empty set of input symbols.

(iii) Y is a finite non-empty set of output symbols.

(iv) W is a weighting space. i.e., weighting space $W = ([0, \infty), \cdot, \max)$, where $\cdot$ is usual multiplication.

(v) $\alpha : Q \times X \times Q \times Y \rightarrow [0, \infty)$ is called a weighted output function.

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***Definition* 6.** Let $\mathscr{M} = (Q, X, Y, W, \alpha)$ be a max weighted finite state mealy machine. Define $\alpha^* : Q \times X^* \times Q \times Y^* \to [0, \infty)$ is defined as follows: $\forall p, q \in Q$, $x \in X^*$, $a \in X$, $y \in Y^*$, $b \in Y$ and $|x| = |y|$.

(i) $\alpha^*(p, a, q, b) = \alpha(p, a, q, b)$

(ii)

$$\alpha^*(p, \lambda, q, \lambda) = \begin{cases} 1, & \text{if } p = q \\ 0, & \text{if } p \neq q \end{cases}$$

(iii) $\alpha^*(p, a, q, \lambda) = \alpha^*(p, \lambda, q, b) = 0$ and

(iv) $\alpha^*(p, xa, q, yb) = \max\limits_{r \in Q} \{\alpha^*(p, x, r, y) \cdot \alpha(r, a, q, b)\}$

***Lemma* 1.** Let $\mathscr{M} = (Q, X, Y, W, \alpha)$ be a mwfmm. Then $\forall p, q \in Q$, $x \in X^*, y \in Y^*$ if $|x| \neq |y|$ then $\alpha^*(p, x, q, y) = 0$.

***Lemma* 2.** Let $\mathscr{M} = (Q, X, Y, W, \alpha)$ be a mwfmm. Then $\alpha^*(p, xu, q, yv) = \max\limits_{r \in Q} \{\alpha^*(p, x, r, y) \cdot \alpha^*(r, u, q, v)\}$, $\forall p, q \in Q$, $x, u \in X^*$, $y, v \in Y^*$ and if $|x| = |y|$; $|u| = |v|$.

4 Homomorphism between two Max weighted finite state mealy machines

weak homomorphism, homomorphism and strong homomorphism [4] on $mwfmm$ is established in this section.

***Definition* 7.** Let $\mathscr{M}_1 = (Q_1, X_1, Y_1, W, \alpha_1)$ and $\mathscr{M}_2 = (Q_2, X_2, Y_2, W, \alpha_2)$ be two $mwfmms$. Let $\tau : Q_1 \to Q_2$ be a function and let $f : X_1 \to X_2$, $g : Y_1 \to Y_2$ be one-one functions. Then

(a) $(\tau, f, g) : \mathscr{M}_1 \to \mathscr{M}_2$ is a weak homomorphism if $\forall p \in Q_1$, $a \in X_1$, $b \in Y_1$, $\exists q \in Q_1, \alpha_1(p, a, q, b) > 0 \Longrightarrow \exists r \in \tau(Q_1)$, $\alpha_2\big(\tau(p), f(a), r, g(b)\big) > 0$ and $\exists r \in \tau(Q_1)$, $\alpha_2\big(\tau(p), f(a), r, g(b)\big) > 0 \Longrightarrow \exists q \in Q_1$, $\alpha_1(p, a, q, b) > 0$.

(b) $(\tau, f, g) : \mathscr{M}_1 \to \mathscr{M}_2$ is a homomorphism if $\forall p \in Q_1$, $a \in X_1$, $b \in Y_1$, $\exists q \in Q_1$, $\alpha_1(p, a, q, b) > 0 \Longrightarrow \alpha_2\big(\tau(p), f(a), \tau(q), g(b)\big) > 0$ and $\exists r \in \tau(Q_1)$, $\alpha_2\big(\tau(p), f(a), r, g(b)\big) > 0 \Longrightarrow \exists q \in Q_1$, $\tau(q) = r$, $\alpha_1(p, a, q, b) > 0$.

(c) $(\tau, f, g) : \mathscr{M}_1 \to \mathscr{M}_2$ is a strong homomorphism if $\forall p, q \in Q_1$, $a \in X_1$, $b \in Y_1$, $\alpha_2\big(\tau(p), f(a), \tau(q), g(b)\big) = \max\limits_{t \in Q_1, \tau(t) = \tau(q)} \Big\{\alpha_1(p, a, t, b)\Big\}$.

***Definition* 8.** Let $\mathscr{M}_1 = (Q_1, X_1, Y_1, W, \alpha_1)$ and $\mathscr{M}_2 = (Q_2, X_2, Y_2, W, \alpha_2)$ be two $mwfmms$. and let $(\tau, f, g) : \mathscr{M}_1 \to \mathscr{M}_2$ be a weak homomorphism (homomorphism, strong homomorphism).
Define $f^* : X_1^* \to X_2^*$ as a natural extension of f. That is,

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(i) $f^*(\lambda) = \lambda$

(ii) $f^*(xa) = f^*(x)f(a),\ \forall x \in X_1^*,\ a \in X_1$.

Similarly, define $g^* : Y_1^* \to Y_2^*$ as a natural extension of g. That is,

(i) $g^*(\lambda) = \lambda$

(ii) $g^*(yb) = g^*(y)g(b),\ \forall y \in Y_1^*,\ b \in Y_1$.

Lemma 3. Let $\mathscr{M}_1 = (Q_1, X_1, Y_1, W, \alpha_1)$ and $\mathscr{M}_2 = (Q_2, X_2, Y_2, W, \alpha_2)$ be two $mwfmms$. Let $(\tau, f, g) : \mathscr{M}_1 \to \mathscr{M}_2$ be a weak homomorphism (homomorphism, strong homomorphism), then

(i) $f^*(uv) = f^*(u)f^*(v),\ \forall u, v \in X_1^*$ and

(ii) $g^*(u'v') = g^*(u')g^*(v'),\ \forall u', v' \in Y_1^*$.

Theorem 2. Let $\mathscr{M}_1 = (Q_1, X_1, Y_1, W, \alpha_1)$ and $\mathscr{M}_2 = (Q_2, X_2, Y_2, W, \alpha_2)$ be two $mwfmms$. Let $(\tau, f, g) : \mathscr{M}_1 \to \mathscr{M}_2$ be a homomorphism. Then $\forall p \in Q_1,\ x \in X_1^*,\ y \in Y_1^*$,

(i) $\exists q \in Q_1,\ \alpha_1^*(p, x, q, y) > 0 \Longrightarrow \alpha_2^*\big(\tau(p), f^*(x), \tau(q), g^*(y)\big) > 0$.

(ii) If τ is onto then, $\exists r \in Q_2$,
$\alpha_2^*\big(\tau(p), f^*(x), r, g^*(y)\big) > 0 \Longrightarrow \exists q \in Q_1,\ \tau(q) = r,\ \alpha_1^*(p, x, q, y) > 0$.

Lemma 4. Let $\mathscr{M}_1 = (Q_1, X_1, Y_1, W, \alpha_1)$ and $\mathscr{M}_2 = (Q_2, X_2, Y_2, W, \alpha_2)$ be two $mwfmms$ and let $(\tau, f, g) : \mathscr{M}_1 \to \mathscr{M}_2$ be a strong homomorphism. Then $\forall p, r \in Q_1,\ a \in X_1,\ b \in Y_1$, if $\alpha_2\big(\tau(p), f(a), \tau(r), g(b)\big) > 0$, then $\exists t \in Q_1$ such that $\alpha_1(p, a, t, b) > 0$ and $\tau(t) = \tau(r)$. Further, $\forall p \in Q_1$, if $\tau(p) = \tau(q)$, then $\alpha_1(p, a, t, b) \geq \alpha_1(q, a, r, b)$.

Theorem 3. Let $\mathscr{M}_1 = (Q_1, X_1, Y_1, W, \alpha_1)$ and $\mathscr{M}_2 = (Q_2, X_2, Y_2, W, \alpha_2)$ be two $mwfmms$ and let $(\tau, f, g) : \mathscr{M}_1 \to \mathscr{M}_2$ be a strong homomorphism. Then τ is one-one if and only if $\forall p, q \in Q_1,\ x \in X_1^*,\ y \in Y_1^*$,
$\alpha_1^*(p, x, q, y) = \alpha_2^*\big(\tau(p), f^*(x), \tau(q), g^*(y)\big)$.

5 Identical states

In this section, equivalence relation related to weakly identical (identical, strongly identical) states on $mwfmm$ is investigated.

Definition 9. Let $\mathscr{M} = (Q, X, Y, W, \alpha)$ be a $mwfmm$. Two states $p_1,\ p_2 \in Q$ are said to be weakly identical, written $p_1 \simeq p_2$, if $\forall a \in X,\ b \in Y,\ \exists q \in Q,\ \alpha(p_1, a, q, b) > 0 \Longleftrightarrow \exists r \in Q,\ \alpha(p_2, a, r, b) > 0$.

Lemma 5. Let $\mathscr{M} = (Q, X, Y, W, \alpha)$ be a $mwfmm$ and a relation $\simeq$ as defined above. Then $\simeq$ is an equivalence relation on Q.

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Proof: Let $p_1 \in Q$, then $\forall a \in X,\ b \in Y$,
$\exists q \in Q,\ \alpha(p_1, a, q, b) > 0 \iff \exists r \in Q,\ \alpha(p_1, a, r, b) > 0$. Thus by selecting r as q, implies that $p_1 \simeq p_1$. Therefore $\simeq$ is reflexive.
Let $p_1,\ p_2 \in Q$, if $p_1 \simeq p_2$, then if $\forall a \in X,\ b \in Y$,
$\exists q \in Q,\ \alpha(p_1, a, q, b) > 0 \iff \exists r \in Q,\ \alpha(p_2, a, r, b) > 0$, implies that
$\forall a \in X,\ b \in Y,\ \exists r \in Q,\ \alpha(p_2, a, r, b) > 0 \iff \exists q \in Q,\ \alpha(p_1, a, q, b) > 0$.
Therefore $p_2 \simeq p_1$. Thus $\simeq$ is symmetric.
Let $p_1,\ p_2,\ p_3 \in Q$, if $p_1 \simeq p_2$, then $\forall a \in X,\ b \in Y$,
$\exists q \in Q,\ \alpha(p_1, a, q, b) > 0 \iff \exists r \in Q,\ \alpha(p_2, a, r, b) > 0$ and
if $p_2 \simeq p_3$, then $\forall a \in X,\ b \in Y$,
$\exists r \in Q,\ \alpha(p_2, a, r, b) > 0 \iff \exists s \in Q,\ \alpha(p_3, a, s, b) > 0$, implies that
$\forall a \in X,\ b \in Y,\ \exists q \in Q,\ \alpha(p_1, a, q, b) > 0 \iff \exists s \in Q,\ \alpha(p_3, a, s, b) > 0$.
Therefore $p_1 \simeq p_3$. Hence $\simeq$ is transitive.
Therefore $\simeq$ is an equivalence relation on Q. ■

Definition 10. Let $\mathscr{M} = (Q, X, Y, W, \alpha)$ be a $mwfmm$. Two states $p_1,\ p_2 \in Q$ are said to be identical, written $p_1 \sim p_2$, if one of the following holds:

(i) $p_1 = p_2$.

(ii) $\forall a \in X,\ b \in Y$,
$\exists q \in Q,\ \alpha(p_1, a, q, b) > 0 \Longrightarrow \exists r \in Q,\ \alpha(p_2, a, r, b) > 0$ and $q \sim r$
and
$\exists r \in Q,\ \alpha(p_2, a, r, b) > 0 \Longrightarrow \exists q \in Q,\ \alpha(p_1, a, q, b) > 0$ and $r \sim q$.

Lemma 6. Let $\mathscr{M} = (Q, X, Y, W, \alpha)$ be a $mwfmm$ and a relation $\sim$ as defined above. Then $\sim$ is an equivalence relation on Q.

Proof: Let $p_1 \in Q$, then

(i) $p_1 = p_1$.

(ii) $\forall a \in X,\ b \in Y$,
$\exists q \in Q,\ \alpha(p_1, a, q, b) > 0 \Longrightarrow \exists r \in Q, \alpha(p_1, a, r, b) > 0$ and $q \sim r$. Thus by selecting r as q.
and
$\exists r \in Q,\ \alpha(p_1, a, r, b) > 0 \Longrightarrow \exists q \in Q,\ \alpha(p_1, a, q, b) > 0$ and $r \sim q$. Thus by selecting q as r.

implies that $p_1 \sim p_1$. Therefore $\sim$ is reflexive.
Let $p_1,\ p_2 \in Q$, if $p_1 \sim p_2$, then either

(i) $p_1 = p_2$. (or)

(ii) $\forall a \in X,\ b \in Y$,
$\exists q \in Q,\ \alpha(p_1, a, q, b) > 0 \Longrightarrow \exists r \in Q,\ \alpha(p_2, a, r, b) > 0$ and $q \sim r$ and
$\exists r \in Q,\ \alpha(p_2, a, r, b) > 0 \Longrightarrow \exists q \in Q,\ \alpha(p_1, a, q, b) > 0$ and $r \sim q$.

implies that

(i) $p_2 = p_1$. (or)

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(ii) $\forall a \in X,\ b \in Y$,
$\exists r \in Q,\ \alpha(p_2, a, r, b) > 0 \Longrightarrow \exists q \in Q,\ \alpha(p_1, a, q, b) > 0$ and $r \sim q$
and
$\exists q \in Q,\ \alpha(p_1, a, q, b) > 0 \Longrightarrow \exists r \in Q,\ \alpha(p_2, a, r, b) > 0$ and $q \sim r$.

Therefore $p_2 \sim p_1$. Thus $\sim$ is symmetric.
Let $p_1,\ p_2,\ p_3 \in Q$. If $p_1 \sim p_2$, then either

(i) $p_1 = p_2$. (or)

(ii) $\forall a \in X,\ b \in Y$,
$\exists q \in Q,\ \alpha(p_1, a, q, b) > 0 \Longrightarrow \exists r \in Q,\ \alpha(p_2, a, r, b) > 0$ and $q \sim r$ and
$\exists r \in Q,\ \alpha(p_2, a, r, b) > 0 \Longrightarrow \exists q \in Q,\ \alpha(p_1, a, q, b) > 0$ and $r \sim q$.

and if $p_2 \sim p_3$, then either

(i) $p_2 = p_3$. (or)

(ii) $\forall a \in X,\ b \in Y$,
$\exists r \in Q,\ \alpha(p_2, a, r, b) > 0 \Longrightarrow \exists s \in Q,\ \alpha(p_3, a, s, b) > 0$ and $r \sim s$ and
$\exists s \in Q,\ \alpha(p_3, a, s, b) > 0 \Longrightarrow \exists r \in Q,\ \alpha(p_2, a, r, b) > 0$ and $s \sim r$.

implies that

(i) $p_1 = p_3$. (or)

(ii) $\forall a \in X,\ b \in Y$,
$\exists q \in Q,\ \alpha(p_1, a, q, b) > 0 \Longrightarrow \exists s \in Q,\ \alpha(p_3, a, s, b) > 0$ and $q \sim s$ and
$\exists s \in Q,\ \alpha(p_3, a, s, b) > 0 \Longrightarrow \exists q \in Q,\ \alpha(p_1, a, q, b) > 0$ and $s \sim q$.

Therefore $p_1 \sim p_3$. Thus $\sim$ is transitive.
Therefore $\sim$ is an equivalence relation on Q. ■

Definition 11. Let $\mathscr{M} = (Q, X, Y, W, \alpha)$ be a $mwfmm$. Two states $p_1,\ p_2 \in Q$ are said to be strongly identical, written $p_1 \equiv p_2$, if one of the following holds:

(i) $p_1 = p_2$.

(ii) $\forall a \in X,\ b \in Y$,
$\exists q \in Q,\ \alpha(p_1, a, q, b) > 0 \Longrightarrow \exists r \in Q,\ \alpha(p_2, a, r, b) \geq \alpha(p_1, a, q, b)$ and $q \equiv r$
and
$\exists r \in Q,\ \alpha(p_2, a, r, b) > 0 \Longrightarrow \exists q \in Q,\ \alpha(p_1, a, q, b) \geq \alpha(p_2, a, r, b)$ and $r \equiv q$.

Lemma 7. Let $\mathscr{M} = (Q, X, Y, W, \alpha)$ be a $mwfmm$ and a relation $\equiv$ as defined above. Then $\equiv$ is an equivalence relation on Q.

Proof: Let $p_1 \in Q$, then

(i) $p_1 = p_1$.

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(ii) $\forall a \in X,\, b \in Y$,
$\exists q \in Q,\, \alpha(p_1,a,q,b) > 0 \implies \exists r \in Q,\, \alpha(p_1,a,r,b) \geq \alpha(p_1,a,q,b)$ and $q \equiv r$. Thus by selecting r as q. and
$\exists r \in Q,\, \alpha(p_1,a,r,b) > 0 \implies \exists q \in Q,\, \alpha(p_1,a,q,b) \geq \alpha(p_1,a,r,b)$ and $r \equiv q$. Thus by selecting q as r.

implies that $p_1 \equiv p_1$. Therefore $\equiv$ is reflexive.
Let $p_1,\, p_2 \in Q$, if $p_1 \equiv p_2$, then either

(i) $p_1 = p_2$. (or)

(ii) $\forall a \in X,\, b \in Y$,
$\exists q \in Q,\, \alpha(p_1,a,q,b) > 0 \implies \exists r \in Q,\, \alpha(p_2,a,r,b) \geq \alpha(p_1,a,q,b)$ and $q \equiv r$ and
$\exists r \in Q,\, \alpha(p_2,a,r,b) > 0 \implies \exists q \in Q,\, \alpha(p_1,a,q,b) \geq \alpha(p_2,a,r,b)$ and $r \equiv q$.

implies that

(i) $p_2 = p_1$. (or)

(ii) $\forall a \in X,\, b \in Y$,
$\exists r \in Q,\, \alpha(p_2,a,r,b) > 0 \implies \exists q \in Q,\, \alpha(p_1,a,q,b) \geq \alpha(p_2,a,r,b)$ and $r \equiv q$
and
$\exists q \in Q,\, \alpha(p_1,a,q,b) > 0 \implies \exists r \in Q,\, \alpha(p_2,a,r,b) \geq \alpha(p_1,a,q,b)$ and $q \equiv r$.

Therefore $p_2 \equiv p_1$. Thus $\equiv$ is symmetric.
Let $p_1,\, p_2,\, p_3 \in Q$. If $p_1 \equiv p_2$, then either

(i) $p_1 = p_2$. (or)

(ii) $\forall a \in X,\, b \in Y$,
$\exists q \in Q,\, \alpha(p_1,a,q,b) > 0 \implies \exists r \in Q,\, \alpha(p_2,a,r,b) \geq \alpha(p_1,a,q,b)$ and $q \equiv r$ and
$\exists r \in Q,\, \alpha(p_2,a,r,b) > 0 \implies \exists q \in Q,\, \alpha(p_1,a,q,b) \geq \alpha(p_2,a,r,b)$ and $r \equiv q$.

and if $p_2 \equiv p_3$, then either

(i) $p_2 = p_3$. (or)

(ii) $\forall a \in X,\, b \in Y$,
$\exists r \in Q,\, \alpha(p_2,a,r,b) > 0 \implies \exists s \in Q,\, \alpha(p_3,a,s,b) \geq \alpha(p_2,a,r,b)$ and $r \equiv s$ and
$\exists s \in Q,\, \alpha(p_3,a,s,b) > 0 \implies \exists r \in Q,\, \alpha(p_2,a,r,b) \geq \alpha(p_3,a,s,b)$ and $s \equiv r$.

implies that

(i) $p_1 = p_3$. (or)

(ii) $\forall a \in X,\, b \in Y,$
$\exists q \in Q,\, \alpha(p_1,a,q,b) > 0 \Longrightarrow \exists s \in Q,\, \alpha(p_3,a,s,b) \geq \alpha(p_1,a,q,b)$ and $q \equiv s$ and
$\exists s \in Q,\, \alpha(p_3,a,s,b) > 0 \Longrightarrow \exists q \in Q,\, \alpha(p_1,a,q,b) \geq \alpha(p_3,a,s,b)$ and $s \equiv q$.

Therefore $p_1 \equiv p_3$. Thus $\equiv$ is transitive.
Therefore $\equiv$ is an equivalence relation on Q. ■

Lemma 8. Let $\mathscr{M} = (Q,X,Y,W,\alpha)$ be a $mwfmm$. Two states $p_1,\, p_2 \in Q$ are identical, if and only if one of the following holds:

(i) $p_1 = p_2$.

(ii) $\forall x \in X^*,\, y \in Y^*,$
$\exists q \in Q,\, \alpha^*(p_1,x,q,y) > 0 \Longrightarrow \exists r \in Q,\, \alpha^*(p_2,x,r,y) > 0$ and $q \sim r$
and
$\exists r \in Q,\, \alpha^*(p_2,x,r,y) > 0 \Longrightarrow \exists q \in Q,\, \alpha^*(p_1,x,q,y) > 0$ and $r \sim q$.

Proof: Let $p_1,\, p_2 \in Q$ be such that $p_1 \sim p_2$.
We Know that a relation '$\sim$' is symmetry, in order to prove the 'if' part, all we have to prove is the following: $\forall x \in X^*,\, y \in Y^*,$
$\exists q \in Q,\, \alpha^*(p_1,x,q,y) > 0 \Longrightarrow \exists r \in Q,\, \alpha^*(p_2,x,r,y) > 0$ and $r \sim q$.
Let $x \in X^*,\, y \in Y^*,\, q \in Q$ and $\alpha^*(p_1,x,q,y) > 0$.
we prove this result by induction on $|x| = n$.
If $n = 0$, then $x = y = \lambda$.
Therefore $\alpha^*(p_1,x,q,y) = \alpha^*(p_1,\lambda,q,\lambda) > 0$
implies $p_1 = q$ and $\alpha^*(p_1,\lambda,p_1,\lambda) = 1$.
Now $\alpha^*(p_2,\lambda,p_2,\lambda) > 0$. Further $p_1 \sim p_2$. Thus by selecting r as p_2.
Therefore the result is true for $n = 0$.
suppose the result is true for all $u \in X^*$ such that $|u| = n-1, n > 0$.
Let $x = ua,\, y = vb$, where $u \in X^*,\, a \in X,\, v \in Y^*,\, b \in Y$
and $|u| = |v| = n-1$. Now

$$\begin{aligned}\alpha^*(p_1,x,q,y) &= \alpha^*(p_1,ua,q,vb)\\ &= \max_{s \in Q}\left\{\alpha^*(p_1,u,s,v)\cdot\alpha(s,a,q,b)\right\}\end{aligned}$$

Since Q is finite, $\exists t \in Q$ such that
$\alpha^*(p_1,u,t,v)\cdot\alpha(t,a,q,b) = \max_{s \in Q}\left\{\alpha^*(p_1,u,s,v)\cdot\alpha(s,a,q,b)\right\} > 0.$
Thus $\alpha^*(p_1,u,t,v) > 0$ and $\alpha(t,a,q,b) > 0$.
By the induction hypothesis, $\exists t^{'} \in Q$ such that $\alpha^*(p_2,u,t^{'},v) > 0$ and $t^{'} \sim t$.
Now $\alpha(t,a,q,b) > 0$ and $t^{'} \sim t$.
Hence by the induction hypothesis, $\exists r \in Q$ such that $\alpha(t^{'},a,r,b) > 0$ and $r \sim q$.
Thus $\exists r \in Q$ such that $\alpha^*(p_2,u,t^{'},v)\cdot\alpha(t^{'},a,r,b) > 0$ and $r \sim q$.
Thus $\alpha^*(p_2,ua,r,vb) > 0$ and $r \sim q$.
Thus $\alpha^*(p_2,x,r,y) > 0$ and $r \sim q$.
The converse is trivial. ■

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***Lemma* 9.** Let $\mathscr{M}= (Q, X, Y, W, \alpha)$ be a $mwfmm$. Two states p_1, $p_2 \in Q$ are strongly identical, if and only if one of the following holds:

(i) $p_1 = p_2$.

(ii) $\forall x \in X^*$, $y \in Y^*$,
$\exists q \in Q$, $\alpha^*(p_1, x, q, y) > 0 \Longrightarrow \exists r \in Q$, $\alpha^*(p_2, x, r, y) \geq \alpha^*(p_1, x, q, y)$ and $q \equiv r$
and
$\exists r \in Q$, $\alpha^*(p_2, x, r, y) > 0 \Longrightarrow \exists q \in Q$, $\alpha^*(p_1, x, q, y) \geq \alpha^*(p_2, x, r, y)$ and $r \equiv q$.

Proof: Let p_1, $p_2 \in Q$ be such that $p_1 \equiv p_2$.
We Know that a relation '$\equiv$' is symmetry, in order to prove the 'if' part, all we have to prove is the following: $\forall x \in X^*$, $y \in Y^*$,
$\exists q \in Q$, $\alpha^*(p_1, x, q, y) > 0 \Longrightarrow \exists r \in Q$, $\alpha^*(p_2, x, r, y) \geq \alpha^*(p_1, x, q, y)$ and $r \equiv q$.
Let $x \in X^*$, $y \in Y^*$, $q \in Q$ and $\alpha^*(p_1, x, q, y) > 0$.
we prove this result by induction on $|x| = n$.
If $n = 0$, then $x = y = \lambda$.
Therefore $\alpha^*(p_1, x, q, y) = \alpha^*(p_1, \lambda, q, \lambda) > 0$ implies $p_1 = q$
and $\alpha^*(p_1, \lambda, p_1, \lambda) = 1$. Now $\alpha^*(p_1, \lambda, p_1, \lambda) = 1 = \alpha^*(p_2, \lambda, p_2, \lambda)$.
Further $p_1 \equiv p_2$. Thus by selecting r as p_2.
Therefore the result true for $n = 0$.
suppose the result is true for all $u \in X^*$ such that $|u| = n - 1, n > 0$.
Let $x = ua$, $y = vb$ where $u \in X^*$, $a \in X$, $v \in Y^*$, $b \in Y$
and $|u| = |v| = n - 1$.
Now

$$\begin{aligned}\alpha^*(p_1, x, q, y) &= \alpha^*(p_1, ua, q, vb)\\ &= \max_{s\in Q}\left\{\alpha^*(p_1, u, s, v) \cdot \alpha(s, a, q, b)\right\} > 0.\end{aligned}$$

Since Q is finite, $\exists t \in Q$ such that
$\alpha^*(p_1, u, t, v) \cdot \alpha(t, a, q, b) = \max_{s\in Q}\left\{\alpha^*(p_1, u, s, v) \cdot \alpha(s, a, q, b)\right\} > 0$.
Thus $\alpha^*(p_1, u, t, v) > 0$ and $\alpha(t, a, q, b) > 0$.
By the induction hypothesis, $\exists t' \in Q$ such that $\alpha^*(p_2, u, t', v) \geq \alpha^*(p_1, u, t, v)$ and $t' \equiv t$.
Now $\alpha(t, a, q, b) > 0$ and $t' \equiv t$.
Hence by the induction hypothesis, $\exists r \in Q$ such that $\alpha(t', a, r, b) \geq \alpha(t, a, q, b)$ and $r \equiv q$.
Thus $\exists r \in Q$ such that $\alpha^*(p_2, u, t', v) \cdot \alpha(t', a, r, b) \geq \alpha^*(p_1, u, t, v) \cdot \alpha(t, a, q, b)$ and $r \equiv q$.
Thus

$$\begin{aligned}\alpha^*(p_1, x, q, y) &= \alpha^*(p_1, ua, q, vb)\\ &= \max_{s\in Q}\left\{\alpha^*(p_1, u, s, v) \cdot \alpha(s, a, q, b)\right\}\\ &= \alpha^*(p_1, u, t, v) \cdot \alpha(t, a, q, b)\\ &\leqslant \alpha^*(p_2, u, t', v) \cdot \alpha(t', a, r, b)\\ &\leqslant \alpha^*(p_2, ua, r, vb)\\ &\leqslant \alpha^*(p_2, x, r, y) \text{ and } r \equiv q.\end{aligned}$$

Therefore $\alpha^*(p_2, x, r, y) \geq \alpha^*(p_1, x, q, y)$ and $r \equiv q$.
The converse is trivial. ■

6 Admissible Relations

Admissible relations on the set of states of a $(mwfmm)$ is characterized.

Definition 12. Let $\mathscr{M} = (Q, X, Y, W, \alpha)$ be a $mwfmm$ and let $\approx$ be an equivalence relation on Q. Then

1. The equivalence relation $\approx$ is said to be weak admissible if and only if $\forall p, q, s \in Q$, $\forall a \in X$, $\forall b \in Y$, if $p \approx q$ and $\alpha(p, a, s, b) > 0$, then $\exists t \in Q$ such that $\alpha(q, a, t, b) > 0$.

2. The equivalence relation $\approx$ is said to be admissible if and only if $\forall p, q, s \in Q$, $\forall a \in X$, $\forall b \in Y$, if $p \approx q$ and $\alpha(p, a, s, b) > 0$, then $\exists t \in Q$ such that $\alpha(q, a, t, b) > 0$ and $t \approx s$.

3. The equivalence relation $\approx$ is said to be strong admissible if and only if $\forall p, q, s \in Q$, $\forall a \in X$, $\forall b \in Y$, if $p \approx q$ and $\alpha(p, a, s, b) > 0$, then $\exists t \in Q$ such that $\alpha(q, a, t, b) \geq \alpha(p, a, s, b)$ and $t \approx s$.

Lemma 10. Let $\mathscr{M} = (Q, X, Y, W, \alpha)$ be a $mwfmm$ and let $\approx$ be an equivalence relation on Q. Then the equivalence relation $\approx$ is admissible if and only if $\forall p, q, s \in Q$, $\forall u \in X^*$, $\forall v \in Y^*$, if $p \approx q$ and $\alpha^*(p, u, s, v) > 0$, then $\exists t \in Q$ such that $\alpha^*(q, u, t, v) > 0$ and $t \approx s$.

Proof: Suppose $\approx$ is admissible.
Let $p, q \in Q$, $p \approx q$.
Let $u \in X^*$, $v \in Y^*$, $s \in Q$ and suppose $\alpha^*(p, u, s, v) > 0$.
we prove this result by induction on $|u| = n$.
If $n = 0$, then $u = v = \lambda$.
Therefore $\alpha^*(p, u, s, v) = \alpha^*(p, \lambda, s, \lambda) > 0$, implies $p = s$ and $\alpha^*(p, \lambda, p, \lambda) = 1$. Now $\alpha^*(q, \lambda, q, \lambda) = 1 > 0$. Further, $p \approx q$. Thus by selecting t as q, the result holds for the base case.
suppose the result is true for all $w \in X^*$ such that $|w| = n - 1, n > 0$.
Let $u = wa$, $v = zb$ where $w \in X^*$, $a \in X$, $z \in Y^*$, $b \in Y$ and $|w| = |z| = n - 1$.
Now

$$\begin{aligned}\alpha^*(p, u, s, v) &= \alpha^*(p, wa, s, zb)\\ &= \max_{r \in Q} \left\{\alpha^*(p, w, r, z) \cdot \alpha(r, a, s, b)\right\} > 0.\end{aligned}$$

Since Q is finite, $\exists r' \in Q$ such that
$\alpha^*(p, w, r', z) \cdot \alpha(r', a, s, b) = \max_{r \in Q} \left\{\alpha^*(p, w, r, z) \cdot \alpha(r, a, s, b)\right\} > 0$.
Thus $\alpha^*(p, w, r', z) > 0$ and $\alpha(r', a, s, b) > 0$.
By the induction hypothesis, $\exists t' \in Q$ such that $\alpha^*(q, w, t', z) > 0$ and $t' \approx r'$.
Now $\alpha(r', a, s, b) > 0$ and $t' \approx r'$.

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Hence by the induction hypothesis, $\exists t \in Q$ such that $\alpha(t^{'}, a, t, b) > 0$ and $t \approx s$.
Thus $\exists t \in Q$ such that $\alpha^*(q, w, t^{'}, z) \cdot \alpha(t^{'}, a, t, b) > 0$ and $t \approx s$.
Thus $\alpha^*(q, wa, t, zb) > 0$ and $t \approx s$. Thus $\alpha^*(q, u, t, v) > 0$ and $t \approx s$.
The converse is trivial. ■

Lemma 11. Let $\mathscr{M} = (Q, X, Y, W, \alpha)$ be a $mwfmm$ and let $\approx$ be an equivalence relation on Q. Then the equivalence relation $\approx$ is strong admissible if and only if $\forall p, q, s \in Q$, $\forall u \in X^*$, $\forall v \in Y^*$, if $p \approx q$ and $\alpha^*(p, u, s, v) > 0$, then $\exists t \in Q$ such that $\alpha^*(q, u, t, v) \geq \alpha^*(p, u, s, v)$ and $t \approx s$.

Proof: Suppose $\approx$ is strong admissible.
Let $p, q \in Q$, $p \approx q$.
Let $u \in X^*$, $v \in Y^*$, $s \in Q$ and suppose $\alpha^*(p, u, s, v) > 0$.
we prove this result by induction on $|u| = n$.
If $n = 0$, then $u = v = \lambda$.
Therefore $\alpha^*(p, u, s, v) = \alpha^*(p, \lambda, s, \lambda) > 0$, implies $p = s$
and $\alpha^*(p, \lambda, p, \lambda) = 1$.
Now $\alpha^*(p, \lambda, p, \lambda) = 1 = \alpha^*(q, \lambda, q, \lambda)$ [since $p \approx q$]. Thus by selecting t as q, the result holds for the base case.
suppose the result is true for all $w \in X^*$ such that $|w| = n - 1, n > 0$.
Let $u = wa$, $v = zb$ where $w \in X^*$, $a \in X$, $z \in Y^*$, $b \in Y$ and
$|w| = |z| = n - 1$.
Now

$$\begin{aligned}\alpha^*(p, u, s, v) &= \alpha^*(p, wa, s, zb) \\ &= \max_{r \in Q} \left\{\alpha^*(p, w, r, z) \cdot \alpha(r, a, s, b)\right\} > 0.\end{aligned}$$

Since Q is finite, $\exists r^{'} \in Q$ such that
$\alpha^*(p, w, r^{'}, z) \cdot \alpha(r^{'}, a, s, b) = \max_{r \in Q} \left\{\alpha^*(p, w, r, z) \cdot \alpha(r, a, s, b)\right\} > 0.$
Thus $\alpha^*(p, w, r^{'}, z) > 0$ and $\alpha(r^{'}, a, s, b) > 0$.
By the induction hypothesis, $\exists t^{'} \in Q$ such that $\alpha^*(q, w, t^{'}, z) \geq \alpha^*(p, w, r^{'}, z)$ and $t^{'} \approx r^{'}$.
Now $\alpha(r^{'}, a, s, b) > 0$ and $t^{'} \approx r^{'}$.
Hence by the induction hypothesis, $\exists t \in Q$ such that $\alpha(t^{'}, a, t, b) \geq \alpha(r^{'}, a, s, b)$ and $t \approx s$.
Thus $\exists t \in Q$ such that $\alpha^*(q, w, t^{'}, z) \cdot \alpha(t^{'}, a, t, b) \geq \alpha^*(p, w, r^{'}, z) \cdot \alpha(r^{'}, a, s, b)$ and $t \approx s$.
Thus

$$\begin{aligned}\alpha^*(p, u, s, v) &= \alpha^*(p, wa, s, zb) \\ &= \max_{r \in Q} \left\{\alpha^*(p, w, r, z) \cdot \alpha(r, a, s, b)\right\} \\ &= \alpha^*(p, w, r^{'}, z) \cdot \alpha(r^{'}, a, s, b) \\ &\leqslant \alpha^*(q, w, t^{'}, z) \cdot \alpha(t^{'}, a, t, b) \\ &\leqslant \alpha^*(q, wa, t, zb) \\ &\leqslant \alpha^*(q, u, t, v)\end{aligned}$$

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Therefore $\alpha^*(q, u, t, v) \geq \alpha^*(p, u, s, v)$ and $t \approx s$.
The converse is trivial. ■

Theorem 4. Let $\mathscr{M}= (Q, X, Y, W, \alpha)$ be a $mwfmm$ and let $\simeq$, $\sim$, $\equiv$ be an equivalence relation on Q. Then $\simeq$, ($\sim$, $\equiv$) is a weak admissible (admissible, strong admissible) relation on Q.

Proof: Let p_1, $p_2 \in Q$, if $p_1 \simeq p_2$, then if $\forall a \in X$, $b \in Y$,
$\exists q \in Q$, $\alpha(p_1, a, q, b) > 0 \iff \exists r \in Q$, $\alpha(p_2, a, r, b) > 0$, which implies that $p_1 \approx p_2$.
Therefore '$\simeq$' is a is a weak admissible relation on Q.
The proofs of the other cases are similar. ■

Theorem 5. Let $\mathscr{M}= (Q, X, Y, W, \alpha)$ be a $mwfmm$ and let $\approx$ is a weak admissible (admissible, strong admissible) relation on Q. Then $\approx$ is a refinement of $\simeq$, ($\sim$, $\equiv$).

Proof: Let $\approx$ be a weak admissible relation on Q.
By definition of weak admissible, $\forall p, q, s \in Q$, $\forall a \in X$, $\forall b \in Y$, if $p \approx q$ and $\alpha(p, a, s, b) > 0$, then $\exists t \in Q$ such that $\alpha(q, a, t, b) > 0$, which implies that $p \simeq q$.
Therefore $\approx$ is a refinement of $\simeq$.
The proofs of the other cases are similar. ■

Definition 13. Let $\mathscr{M}_1= (Q_1, X, Y, W, \alpha_1)$ and $\mathscr{M}_2 = (Q_2, X, Y, W, \alpha_2)$ be two $mwfmms$. Let $\gamma : \mathscr{M}_1 \to \mathscr{M}_2$ be either a weak homomorphism (homomorphism, strong homomorphism). The kernel of γ, denoted by $ker(\gamma)$ is defined as

$$ker(\gamma) = \{(p_1, q_1)|\gamma(p_1) = \gamma(q_1)\}$$

Lemma 12. Let $\mathscr{M}_1= (Q_1, X, Y, W, \alpha_1)$ and $\mathscr{M}_2 = (Q_2, X, Y, W, \alpha_2)$ be two $mwfmms$ and let $\gamma : \mathscr{M}_1 \to \mathscr{M}_2$ be a weak homomorphism. Then $ker(\gamma)$ is a weak admissible relation on Q.

Proof: Suppose γ is a weak homomorphism.
By definition, $ker(\gamma) = \{(p_1, q_1)|\gamma(p_1) = \gamma(q_1)\}$.
We show that $ker(\gamma)$ is an equivalence relation on Q.
Let $p_1 \in Q_1$, then $\gamma(p_1) = \gamma(p_1)$. Therefore $(p_1, p_1) \in ker(\gamma)$.
Thus $ker(\gamma)$ is reflexive.
Let $(p_1, q_1) \in ker(\gamma)$. Then $\gamma(p_1) = \gamma(q_1)$, implies that $\gamma(q_1) = \gamma(p_1)$. Therefore $(q_1, p_1) \in ker(\gamma)$. Thus $ker(\gamma)$ is symmetric.
Let (p_1, q_1), $(q_1, r_1) \in ker(\gamma)$ implies that $\gamma(p_1) = \gamma(q_1)$ and $\gamma(q_1) = \gamma(r_1)$.
Then we have $\gamma(p_1) = \gamma(q_1) = \gamma(r_1)$, implies that $\gamma(p_1) = \gamma(r_1)$.
Therefore $(p_1, r_1) \in ker(\gamma)$.
Thus $ker(\gamma)$ is transitive.
Therefore $ker(\gamma)$ is an equivalence relation on Q.
Let p_1, $q_1 \in Q_1$ and $(p_1, q_1) \in ker(\gamma)$. Then $\gamma(p_1) = \gamma(q_1)$.
Let $a \in X$, $b \in Y$, $q \in Q_1$.
Suppose $\alpha_1(p_1, a, q, b) > 0$.
Then $\alpha_2\big(\gamma(q_1), a, \gamma(q), b)\big) = \alpha_2\big(\gamma(p_1), a, \gamma(q), b)\big) > 0$.
Thus $\alpha_1(q_1, a, q, b) > 0$ and hence $ker(\gamma)$ is a weak admissible relation on Q. ■

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***Lemma* 13.** Let $\mathscr{M}_1 = (Q_1, X, Y, W, \alpha_1)$ and $\mathscr{M}_2 = (Q_2, X, Y, W, \alpha_2)$ be two $mwfmms$ and let $\gamma : \mathscr{M}_1 \to \mathscr{M}_2$ be a homomorphism. Then $ker(\gamma)$ is an admissible relation on Q.

Proof: Suppose γ is a homomorphism.
Clearly $ker(\gamma)$ is an equivalence relation on Q.
Let p_1, $q_1 \in Q_1$ and $(p_1, q_1) \in ker(\gamma)$. Then $\gamma(p_1) = \gamma(q_1)$.
Let $a \in X$, $b \in Y$, $s \in Q_1$.
Suppose $\alpha_1(p_1, a, s, b) > 0$.
Then $\alpha_2\big(\gamma(q_1), a, \gamma(s), b)\big) = \alpha_2\big(\gamma(p_1), a, \gamma(s), b)\big) > 0$.
Thus $\alpha_1(q_1, a, s, b) > 0$. Since $\gamma(s) = \gamma(s)$, $(s, s) \in ker(\gamma)$.
Thus $ker(\gamma)$ is an admissible relation on Q. ■

***Lemma* 14.** Let $\mathscr{M}_1 = (Q_1, X, Y, W, \alpha_1)$ and $\mathscr{M}_2 = (Q_2, X, Y, W, \alpha_2)$ be two $mwfmms$ and let $\gamma : \mathscr{M}_1 \to \mathscr{M}_2$ be a strong homomorphism. Then $ker(\gamma)$ is a strong admissible relation on Q.

Proof: Suppose γ is a strong homomorphism.
Clearly $ker(\gamma)$ is an equivalence relation on Q.
Let p_1, $q_1 \in Q_1$ and $(p_1, q_1) \in ker(\gamma)$. Then $\gamma(p_1) = \gamma(q_1)$.
Let $a \in X$, $b \in Y$, $r \in Q_1$.
Suppose $\alpha_1(p_1, a, r, b) > 0$.
Then $\alpha_2\big(\gamma(q_1), a, \gamma(r), b)\big) = \alpha_2\big(\gamma(p_1), a, \gamma(r), b)\big) \geq \alpha_1(p_1, a, r, b) > 0$. By lemma 4 $\exists t \in Q_1$, such that $\alpha_1(q_1, a, t, b) \geq \alpha_1(p_1, a, r, b) > 0$ and
$\gamma(t) = \gamma(r)$. Since $\gamma(t) = \gamma(r)$, $(t, r) \in ker(\gamma)$.
Thus $ker(\gamma)$ is a strong admissible relation on Q. ■

7 conclusion

In this paper an attempt is made to introduce the algebraic concepts such as homomorphism and admissible relation on Max weighted finite state mealy machine. These concepts can be extended towards other algebraic structures which will give fruitful results in the area of weighted automata.

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